Astronomy 722: Radiative Transfer and Gas Dynamics

San Francisco State University ©2018 Andisheh Mahdavi Spring 2018 Tu Th 5:10PM

Homework 2 Due 5:10PM 2/20

While I may have consulted with other students in the class regarding this homework, the solutions presented here are my own work. I understand that to get full credit, I have to show all the steps necessary to arrive at the answer, and unless it is obvious, explain my reasoning using diagrams and/or complete sentences.

Name  Signature:

1. (50%) Imagine a spherically symmetric, constant density cloud of dust of mass $M$ and radius $R$. The cloud has a star at the center. The star is a blackbody of temperature $T$ and is much, much smaller than the cloud of dust. The dust has no emission, but does absorb the star’s light with known frequency-dependent opacity $\kappa_\nu = \beta \nu$, where $\beta$ is a positive constant.

I sit with a telescope a distance $D$ from the star and observe its spectrum. I find the source has a smooth spectrum with no absorption lines, no local minima, and with just one peak (global maximum) at frequency $\nu = kT/h$, instead of $2.82kT/h$ as would be expected for an unabsorbed blackbody.

(a) (40%) Using the location of this observed peak in the spectrum, find the relationship between $\beta$, $M$, and $R$. Argue that if we know the precise numerical values for two out of those three quantities, we will be able to infer the third.

(b) (10%) As a sanity check to see if this opacity model is correct, we can try to see if the total dimming of the star follows expectations. How much smaller should we expect the total frequency-integrated flux of this source to be compared to an unabsorbed star of the same type?

N.B. Numerical integration will be required for the above problem.

2. (50%) Similar to the above, but for scattering: consider a star of intrinsic brightness $I^*_\nu$ embedded in a constant density cloud of gas of radius $R$ that is much bigger than it. The cloud does not emit or absorb light, but scatters it with a constant scattering coefficient $\sigma_\nu$. This problem aims to guide you through the process of calculating the resulting scattering output.

(a) (10%) Describe qualitatively what happens to the light within the gas cloud. Argue that once the star has been shining for a long time, and the cloud has reached steady-state, then the radiation field throughout the cloud can be written as $I_\nu(\tau_\nu, \theta)$ where the dimensionless optical depth for scattering is given by

$$d\tau_\nu = -\sigma_\nu dr$$

where $r$ is the distance along the line of sight, and $\theta$ is the angle that the radiation field makes with the radial direction at any point. Show that $\tau_\nu = 0$ is the surface of the cloud closest to us, and $\tau_\nu = \tau^*_\nu \approx R\sigma_\nu$ is the optical depth to the surface of the star.

(b) (10%) Consider the radiation field between us and the star throughout the cloud of gas. Show that the appropriate radiative transfer equations can be written as

$$\mu \frac{\partial I_\nu(\tau_\nu, \mu)}{\partial \tau_\nu} = I_\nu(\tau_\nu, \mu) - J_\nu(\tau_\nu)$$

where $J_\nu(\tau_\nu)$ is the mean intensity and $\mu = \cos \theta$. Argue that this equation only needs to be integrated between $\tau_\nu = \tau^*_\nu$ and 0 when considering the volume between us and the star.
(c) (25%) Guess a solution for the above equation in the form

\[ I_\nu(\mu, \tau) = A\mu + B\tau + C \]

Solve for A, B, and C using the following known facts:

- The precise theoretical relationship between \( J_\nu \) and \( I_\nu \)
- \( I_\nu = I_\nu^\star \) at the surface of the star for outgoing rays
- \( I_\nu = 0 \) at the surface of the cloud for incoming rays (i.e., no light is entering the cloud)

Note that the equation in (b) can be solved exactly for this guess and, unlike the examples in the text, no Eddington or similar approximation is required.

(d) (5%) What fraction of its own light output is shining back on the star?