Physics and Astronomy 340: The Big Bang
San Francisco State University (c) 2017 Andisheh Mahdavi Spring 2017

Homework 2 Due 5:00PM March 2

While I may have consulted with other students in the class regarding this homework, the solutions presented here are my own work. I understand that to get full credit, I have to show all the steps necessary to arrive at the answer, and unless it is obvious, explain my reasoning using diagrams and/or complete sentences.

Name: 
Signature: 

1. (15 points) You and your friend plan trips to Shanghai, China and Manila, the Philippines, for the upcoming June 5 transit of Venus. Explain in detail how you would go about verifying the current value of the astronomical unit during this trip.

(a) (5 points) Sketch a reasonably accurate diagram of the geometry of the transit as viewed from both locations, of the angles and triangles involved.

(b) (5 points) The Sun is about half a degree across. The path of Venus across the face of the Sun will be a straight line, but at different positions on the face of the Sun, depending on your exact location. Let the angular distance between these two paths be called $\theta$.

Given your advance knowledge of the AU, predict what you would measure for $\theta$, the angular separation of the paths of the transits as seen from both Manila and Shanghai.

(c) (5 points) Of course, to actually measure the AU without prior knowledge, you need to measure $\theta$. Traditionally, there was no easy way to measure this $\theta$ directly. Instead, it was inferred by measuring the time it took Venus to cross the face of the Sun.

Imagine you can measure the time it takes Venus to transit across the face of the Sun. You do this in Shanghai and your friend does it in Manila. You obviously come up with two different times, $t_S$ and $t_M$, for the transit. How can you use these two times, together with trigonometry and knowledge of Venus’s orbit, to measure $\theta$?

2. This problem needs to be emailed to me as a Mathematica notebook. According to Kelvin-Helmholtz theory, the Sun contracted from an infinitely large sphere to its present size, radiating the energy away as sunlight. There is no nuclear fusion in this model.

(a) (5 points) Show that the total gravitational energy of the Sun at any given moment is

$$ U = -\frac{3GM^2}{5r} $$

where $M$ is the mass of the sun and $r$ is its radius.

(b) (10 points) Using this model, solve for $r(t)$, the radius of the Sun as a function of time, where $t = 0$ corresponds to the present day. Do this by setting the time derivative of the total gravitational potential energy of the Sun equal to its luminosity. Plug in relevant constants.

(c) (5 points) When does $r = \infty$? This is the (incorrect) time in the past when the Sun was born.

(d) (5 points) How long into the future will the sun continue to radiate under this model?

(e) (10 points) Plot the surface temperature of the Sun as a function of time under Kelvin-Helmholtz theory assuming the Sun is a blackbody, for past and for future.

3. (10 points) Liddle 5.3.
4. (10 points) Liddle 5.5.

5. (10 points) Liddle 5.6.

6. (20 points) Solve the Friedman equation, this time assuming a radiation-dominated universe \( (\rho = \rho_{\gamma}/a(t)^4) \) with nonzero \( k \). Do an exact solution of the Friedman equation (i.e. do not assume, as in 5.5, that the curvature term dominates).