Homework 4 Due 1:10PM Monday, April 22nd

Special office hours: Friday 2-3PM for this homework only

While I may have consulted with other students in the class regarding this homework, the solutions presented here are my own work. I understand that to get full credit, I have to show all the steps necessary to arrive at the answer, and unless it is obvious, explain my reasoning using diagrams and/or complete sentences.

Name: ___________________________  Signature: ___________________________

1. (5 points) Liddle 10.1
2. (10 points) Liddle 10.2
3. (5 points) Liddle 10.3
4. (10 points) Liddle 11.2
5. (10 points) Liddle 12.2
6. (5 points) Liddle 12.3
7. (5 points) Just after nucleosynthesis, the baryons in the universe were 75% hydrogen by mass. What is the fraction of hydrogen if we count the number of nuclei instead of mass?

8. (50 points) This is a warm-up problem for your final paper, so do a good job here. Imagine a simplified version of nucleosynthesis where we start out at t=400 seconds with a 1/8 proton-to-neutron ratio (equation 12.9). In this “pretend” nucleosynthesis, only two things can happen: neutrons can decay into protons, and protons and neutrons can combine to form deuterium. Assume (incorrectly) that the temperature is constant.

Let’s call the fraction of neutrons by mass to be \( X_N(t) \), where you’ll need to calculate \( X_N(400) \). For protons, we have \( X_p(t) \), where again you’ll need to calculate \( X_p(400) \). Finally there is the deuterium fraction by mass, \( X_D(t) \), with initial fraction \( X_D(400) = 0 \) (there is none initially).

The rate of change fraction for neutrons is loss through decay, and loss into deuterium:

\[
\frac{dX_N}{dt} = -\frac{X_N}{882s} - \frac{X_N X_p}{21.3s}
\]

The rate of change fraction for proton is gain through neutron decay, and loss into deuterium:

\[
\frac{dX_p}{dt} = +\frac{X_N}{882s} - \frac{X_N X_p}{21.3s}
\]

The rate of change fraction for deuterium is:

\[
\frac{1}{2} \frac{dX_D}{dt} = +\frac{X_N X_p}{21.3s}
\]

Use NDSolve in Mathematica for this system of three differential equations to find out how long it takes for the elements to “freeze out;” i.e., the fractions to become stable. What is the final fraction of Deuterium? N.B. This answer is wrong because we haven’t taken the many other reactions into account. You will do this for your final paper.