Physics 701: Classical Mechanics

San Francisco State University — Fall 2017, Tu-Th 5:00PM

Homework 1 Due 5:00PM Thursday 8/31

While I may have consulted with other students in the class regarding this homework, the solutions presented here are my own work. I understand that to get full credit, I have to show all the steps necessary to arrive at the answer, and unless it is obvious, explain my reasoning using diagrams and/or complete sentences. Please attach this sheet to your homework.

Name ______________________ **Signature:** ______________________

This assignment does not contain a Mathematica project, but it uses Mathematica as a plotting tool.

1. (20 points) Some vector calculus review in three dimensions. Trial-and-error is OK as a means of solving the problems.
   - (5 points) Find a non-zero function that is the divergence of its own gradient.
   - (5 points) Find a non-constant vector field that has both zero divergence and zero curl. Visualize it in Mathematica with `VectorPlot3D`.
   - (5 points) Find a non-constant vector field with non-zero curl and non-zero divergence, such that the curl is equal to the gradient of the divergence. Visualize it in Mathematica with `VectorPlot3D`.
   - (5 points) Find a non-constant vector field, such that its magnitude squared is equal to its divergence. Visualize it in Mathematica with `VectorPlot3D`.

2. (20 points) A particle of mass m moving in the x direction sees an attractive potential energy function of the form $V(x) = k|x|$ where $k > 0$. If it is released from rest with amplitude x_0, what is its velocity when it passes the origin? Find the period, by reduction to quadrature, for a single complete cycle of oscillation.

3. (20 points) Using conservation of memontum, show that a rocket of mass m travelling with speed v and ejecting propellent with velocity v' obeys the equation

   $$m\frac{dv}{dt} + v'\frac{dm}{dt} = -mg$$

   Imagine a rocket that has a mass m_0 without fuel. If you give it fuel, it can eject 1/100 of its mass every second at $v' = 4.1$ km/s. How much fuel (in units of m_0) does the rocket need to escape the Earth? Neglect air resistance.

4. (40 points) In science fiction, spaceships or space stations frequently rotate to maintain artificial gravity. Imagine a hollow cylinder of radius R, rotating uniformly with period P to produce artificial gravity equal to Earth's gravity. Its inhabitants live on the *inside* the curved part of the cylinder in a protected environment. This problem is intended to help you design such a space station so that gravity "feels" normal.
   - (10 points) **How fast it needs to spin**. Calculate the period P of the rotation required to produce Earth-type gravity as a function of R.

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- (10 points) **Dizziness** Is it feasible to live on such a cylinder without getting dizzy? First come up with a criterion for dizziness while standing still that makes sense to you (it doesn't have to be precise, as long as you justify why you think it is a good criterion). Then, estimate how big the cylinder would have to be so that you don't get dizzy.
- (10 points) **What happens if you drop something.** Imagine you live on this space station. In your own rest frame, you stand at $x = 0$. You drop something from a height $y = h$. Write down the equation for its trajectory in the x,y plane as viewed by yourself in your rest frame. At what x values does it hit the ground? Give limits on the values of R and P that would keep this distance manageable (under h). Use Mathematica to plot some of the possible trajectories in the x-y plane. Hint: use $\sin x \approx x$ for $x << 1$ to simplify the algebra.
- (10 points) **Jumping up** Imagine that on Earth you can jump to a height of 1m. You stand next to a friend and jump up in this space station while your friend watches. Give some values of R and P that would allow you to land back within a meter of where you were when you jumped up. Use Mathematica to plot some of the possible trajectories as viewed by your friend.

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