Physics 701: Classical Mechanics

San Francisco State University

Fall 2017, Tu-Th 5:00PM

Homework 3 Due 5:00PM 9/21

While I may have consulted with other students in the class regarding this homework, the solutions presented here are my own work. I understand that to get full credit, I have to show all the steps necessary to arrive at the answer, and unless it is obvious, explain my reasoning using diagrams and/or complete sentences.

Name **Signature:**

1. **Mathematica Project: This problem is tracked separately from other homework for grading purposes. Please email me your Mathematica .nb file**

   Imagine a bead moving on a surface whose shape is given by $z = x^2 + y^2$. The bead experiences downward gravity, and its weight gives rise to sliding friction with coefficient μ.

   - Write down the Cartesian Lagrangian for this system in the absence of friction. Find all conserved quantities. Are there any "hidden" conserved quantities? If so, switch to an appropriate coordinate system and find them.
   - Now including friction, set up the Cartesian equations for solving this problem using Mathematica and plot some sample trajectories. Be sure to plot `x'[t]` and `y'[t]` to show that the particle is slowing down to almost zero velocity.

     Think about the time it takes for the particle to "stop," i.e. reach a speed less than 1% of its maximum speed for all future time. How does this time depend on the force of gravity and the coefficient of friction? Use at least six different trial runs to answer this question.

   Hint: Friction acts in a direction opposite the velocity unit vector. You can pick out e.g. the X-component of the velocity unit vector using an expression like

   ```
   x'[t]/Sqrt[x'[t]^2+y'[t]^2+z'[t]^2+epsilon]
   ```

   where epsilon is a small number (e.g. 10^{-4}) called the "softening parameter" which allows Mathematica to avoid numerical roundoff error when calculating the velocity direction for very small velocities. Such softening parameters occur frequently in scientific computing, and effectively specfiy the desired accuracy of the result.

2. (40 points) In certain situations, particularly one-dimensional systems, it is possible to incorporate frictional effects without needing to add anything to the right-hand side of the Euler-Lagrange equations. Find the equation of motion for the Lagrangian

   $$L = \frac{e^{\gamma t}}{2}\left(m\dot{q}^2 - kq^2\right)$$

   How would you describe this system? Are there any constant of motion? Suppose a point transformation is made of the form

   $$r = e^{\gamma t/2} q$$

   What is the effective Lagrangian in terms of r? Find the equation of motion for r. What do these results say about the conserved quantities for the system?

3. (60 points) My car's fuel efficiency in miles per gallon is an unknown, decreasing function $f(a)$, where a is the car's net acceleration. If I want accelerate from rest to some velocity $v_{\max}$ over a fixed distance, how should I do it so as to use the least amount of gas? Use variational calculus to work out $v(x)$, the velocity profile as a function of distance, which gives me the best fuel efficiency. Also calculate the velocity and acceleration as a function of time.

   Convince me that your chosen solution is actually a minimum (you could do this by picking a suitable $f(a)$, trying several non-optimal $v(x)$ and showing that they use up more gas than your chosen solution).