

# Physics 725: Special and General Relativity

Thornton 335, San Francisco State University

Fall 2011, MWF 3:10PM

## Homework 2 Due 9/14

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While I may have consulted with other students in the class regarding this homework, the solutions presented here are my own work. I understand that to get full credit, I have to show all the steps necessary to arrive at the answer, and unless it is obvious, explain my reasoning using diagrams and/or complete sentences.

**Name**

**Signature:**

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1. Hartle 3.2.
2. Hartle 3.4.
3. Consider the attractive central force potential  $V(r) = -GMme^{-r/r_0}/r_0$ , where  $M$  is much greater than  $m$  and  $r_0$  is a constant, characteristic radius.
  - (a) Which, if any, of Kepler's three laws are valid for this potential? Back up your answer with mathematical proof as much as possible.
  - (b) Write down the relationship between the speed and radius of a circular orbit for the above potential.
  - (c) Show that for circular orbits around fixed  $M$ , the speed-radius relationship reaches a maximum value at a unique value of  $r > 0$ .
  - (d) Compare with the behavior of the  $V(r)$  above with Newtonian gravity by numerically plotting the speed of a circular orbit versus its radius for both cases on top of each other on the same plot (use  $G = M = r_0 = 1$ ).
  - (e) Both  $V(r)$  and the Newtonian gravity potential  $-GMm/r$  are monotonic functions of  $r$ . Why is it that one of them has a maximum circular velocity for a given  $M$  and the other one is unbounded?
4. Hartle 4.2.
5. Hartle problem 4.3. Using graph paper, construct a very carefully drawn and clearly labeled spacetime diagram by hand, clearly showing the motion of the pole and the location of the barn doors in both the frame of the vaulter and the frame of the barn.
6. Imagine that in *example 4.3* (NOT problem 4.3, EXAMPLE 4.3 on p. 64), Alice and Bob send each other a signal every 5 years, each according to his or her own clock. Each signal travels at the speed of light and reaches the other person. Write down the times each of them logs a signal, i.e. "Alice receives Bob's first signal at  $t=?$  on her clock;" and so on, and likewise for Bob. Each person sends the first signal 5 years following departure (from their respective points of view, of course—relativity of simultaneity and all that). Log all signals for the entire round trip.
7. Derive the Lorentz boost formulas (4.21) from first principles by requiring that they be linear, that the events  $(x = 0, t = 0)$  and  $(x' = 0, t' = 0)$  coincide and that they reduce to the Galilean transformation in the limit of low velocity.
8. Hartle 4.18.
9. Hartle 5.1
10. Hartle 5.2