Physics 725: Special and General Relativity

Thornton 335, San Francisco State University (c) 2011 Andisheh Mahdavi Fall 2011, MWF 3:10PM

Homework 5 Due 10/26

While I may have consulted with other students in the class regarding this homework, the solutions presented here are my own work. I understand that to get full credit, I have to show all the steps necessary to arrive at the answer, and unless it is obvious, explain my reasoning using diagrams and/or complete sentences.

Name **Signature:**

1. Your are a very large (i.e. essentially infinite) distance from a 1 solar mass Schwarzschild black hole. At some time $t < 0$ you release a monochromatic green flare (rest-frame $\lambda = 520$nm) from rest towards the black hole. The flare conducts pure radial infall from (essentially) infinity. At $t = 0$, the flare has progressed some distance towards the black hole, and you observe the flare's color as being $\lambda = 521$nm—not changed by much.

   (a) Solve for the color of light you see as the flare continues falling towards the black hole for $t > 0$. It is not possible to solve for $\lambda(t)$ analytically, so solve for the inverse of it, $t(\lambda)$, and plot that.

   (b) At what t does the object become invisible to the naked eye? What is its coordinate radius r at that moment? You will need to figure out the answer numerically.

   Hints: Remember that the energy measured by any observer is $-\mathbf{p} \cdot \mathbf{u}_{\text{obs}}$, where $\mathbf{p}$ is the photon's 4-momentum and $\mathbf{u}_{\text{obs}}$ is the observer's world-line. If we imagine the green flare is an "observer," it must always "observe" its own color to be 520nm. Also remember that $\boldsymbol{\xi} \cdot \mathbf{p} = (1, 0, 0, 0) \cdot \mathbf{p}$ must be conserved for the photon. Remember that you will need to put Gs and cs in the right place to get real units back.

2. Hartle 9.14. *Hint:* first do the problem in Newtonian mechanics. Then remember that the area element is different in GR than in Newtonian mechanics.

3. Hartle 9.15

4. Hartle 9.16