In the Christoffel symbol handout, for the Lagrangian
\[ L^2 = -g_{\alpha\beta} \frac{dx^\alpha}{d\sigma} \frac{dx^\beta}{d\sigma} \]
we said that the velocity derivative is
\[ \frac{\partial L^2}{\partial \left( \frac{dx^\alpha}{d\sigma} \right)} = -2g_{\alpha\beta} \frac{dx^\beta}{d\sigma} \]
To see where the 2 comes from, let's write out all of \( L^2 \) by hand using ugly brute force:
\[
-L^2 = g_{\alpha\beta} \frac{dx^\alpha}{d\sigma} \frac{dx^\beta}{d\sigma} = g_{00} \frac{dx^0}{d\sigma} \frac{dx^0}{d\sigma} + g_{01} \frac{dx^0}{d\sigma} \frac{dx^1}{d\sigma} + g_{02} \frac{dx^0}{d\sigma} \frac{dx^2}{d\sigma} + g_{03} \frac{dx^0}{d\sigma} \frac{dx^3}{d\sigma} + \\
g_{10} \frac{dx^1}{d\sigma} \frac{dx^0}{d\sigma} + g_{11} \frac{dx^1}{d\sigma} \frac{dx^1}{d\sigma} + g_{12} \frac{dx^1}{d\sigma} \frac{dx^2}{d\sigma} + g_{13} \frac{dx^1}{d\sigma} \frac{dx^3}{d\sigma} + \\
g_{20} \frac{dx^2}{d\sigma} \frac{dx^0}{d\sigma} + g_{21} \frac{dx^2}{d\sigma} \frac{dx^1}{d\sigma} + g_{22} \frac{dx^2}{d\sigma} \frac{dx^2}{d\sigma} + g_{23} \frac{dx^2}{d\sigma} \frac{dx^3}{d\sigma} + \\
g_{30} \frac{dx^3}{d\sigma} \frac{dx^0}{d\sigma} + g_{31} \frac{dx^3}{d\sigma} \frac{dx^1}{d\sigma} + g_{32} \frac{dx^3}{d\sigma} \frac{dx^2}{d\sigma} + g_{33} \frac{dx^3}{d\sigma} \frac{dx^3}{d\sigma} \\
\]
Now try evaluating the derivative with respect to the 0th velocity; the corresponding terms in the sum are
\[
- \frac{\partial L^2}{\partial \left( \frac{dx^0}{d\sigma} \right)} = 2g_{00} \frac{dx^0}{d\sigma} + g_{01} \frac{dx^1}{d\sigma} + g_{02} \frac{dx^2}{d\sigma} + g_{03} \frac{dx^3}{d\sigma} + \\
g_{10} \frac{dx^1}{d\sigma} + 0 + 0 + 0 \\
g_{20} \frac{dx^2}{d\sigma} + 0 + 0 + 0 \\
g_{30} \frac{dx^3}{d\sigma} + 0 + 0 + 0 \\
\]
But we know that the metric is a symmetric matrix, i.e.
\[ g_{\alpha\beta} = g_{\beta\alpha} \]
As a result,
\[
- \frac{\partial L^2}{\partial \left( \frac{dx^0}{d\sigma} \right)} = 2g_{00} \frac{dx^0}{d\sigma} + 2g_{01} \frac{dx^1}{d\sigma} + 2g_{02} \frac{dx^2}{d\sigma} + 2g_{03} \frac{dx^3}{d\sigma} = 2g_{0\beta} \frac{dx^\beta}{d\sigma} \\
\]
And the same goes for all the other coordinates labeled 1, 2, and 3.
Let's emphasize the truly important point here: the "2" is valid only for a symmetric metric. Many of our results on the equations of motion would be invalid were the metric not symmetric.