

(* (c) 2009 Andisheh Mahdavi *)
 (* This Mathematica Notebook calculates the orbit of a particle for the equatorial theta=Pi/2 plane in Hartle's wormhole metric under the assumption t=tau. This assumption, while not strictly correct, gives the correct shape for the orbit (because t is proportional to tau for this line element). See the accompanying file wormhole-correct.nb for the full, correct calculation. *)

```
L = Sqrt[1 - D[r[t], t]^2 - (1 + r[t]^2) D[phi[t], t]^2]
eq1 = -D[D[L, r'[t]], t] + D[L, r[t]];
eq2 = -D[D[L, phi'[t]], t] + D[L, phi[t]];
sol = NDSolve[{eq1 == 0, eq2 == 0, r[0] == 2, r'[0] == -0.5, phi[0] == Pi/4, phi'[0] == 0.099},
  {r, phi}, {t, 0, 20}, SolveDelayed -> True]
Plot[r[t] /. sol[[1]], {t, 0, 20}]
g1 = ParametricPlot3D[Evaluate[
  {Sqrt[r[t]^2 + 1] Cos[phi[t]], Sqrt[r[t]^2 + 1] Sin[phi[t]], ArcSinh[r[t]]} /. sol[[1]],
  {t, 0, 20}, AspectRatio -> 1, PlotRange -> All, PlotStyle -> Thickness[0.01]];
g2 = ParametricPlot3D[{Sqrt[(1 + r^2)] Cos[p], Sqrt[(1 + r^2)] Sin[p], ArcSinh[r]},
  {r, -4, 4}, {p, 0, 2 Pi}];
Show[
  g1,
  g2]
```

$$\sqrt{1 - (1 + r[t]^2) \phi'[t]^2 - r'[t]^2}$$

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{r -> InterpolatingFunction[{{0., 20.}}, <>], phi -> InterpolatingFunction[{{0., 20.}}, <>]}
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