

(* (c) 2009 Andisheh Mahdavi *)
 (* This Mathematica notebook shows the exact differential equations for the evolution of the t and r coordinates in Hartle's wormhole metric. Can you complete it so as to recover and plot the correct motion in the wormhole metric, using the commands in wormhole-approximate.nb as your guides? Only the first two differential equations are explicitly calculated, so you will have to fill in the code for phi[tau], and you will have to set theta[sig]=Pi/2, theta'[sig]=0 in order to match the metric in the wormhole-approximate.nb notebook. You will need two initial conditions per parameter---for example, r[0]=2, r'[0]=-0.5 for r. What are sensible initial conditions for t[0] and t'[0]? *)
 Lsq = -t'[sig]^2 + r'[sig]^2 + (r[sig]^2 + 1) (theta'[sig]^2 + Sin[theta[sig]]^2 phi'[sig]^2)
 eq1 = -D[D[Lsq, t'[sig]] /. sig -> tau, tau] + D[Lsq, t[sig]] /. sig -> tau
 eq2 = -D[D[Lsq, r'[sig]] /. sig -> tau, tau] + D[Lsq, r[sig]] /. sig -> tau

$$r'[sig]^2 - t'[sig]^2 + (1 + r[sig]^2) (\text{Sin}[\text{theta}[sig]]^2 \text{phi}'[sig]^2 + \text{theta}'[sig]^2)$$

$$2 t''[\text{tau}]$$

$$2 r[\text{tau}] (\text{Sin}[\text{theta}[\text{tau}]]^2 \text{phi}'[\text{tau}]^2 + \text{theta}'[\text{tau}]^2) - 2 r''[\text{tau}]$$