Motion is Relative

- The place from which motion is observed and measured is a **frame of reference**.
  - An object may have different velocities relative to different frames of reference.
- To measure the speed of an object, we first choose a frame of reference, then we measure the distance per unit time the object moves relative to the frame of reference.
- Isn’t there some reference frame that is still? Isn’t space itself still, and can’t measurements be made relative to still space? -- NO
Postulates of Special Theory of Relativity

- All laws of nature are the same in all uniformly moving frames of reference.
- The speed of light in free space has the same measured value for all observers, regardless of the motion of the source or the motion of the observer; that is, the speed of light is a constant.

Simultaneity

- We say that two events are simultaneous if they occur at the same time.
- Two events that are simultaneous in one frame of reference need not be simultaneous in a frame moving relative to the first frame.
Simultaneity

From the point of view of the observer who travels with the compartment, light from the source travels equal distances to both ends of the compartment and therefore strikes both ends simultaneously.

The events of light striking the front and back of the compartment are not simultaneous from the point of view of an observer in a different frame of reference. Because of the ship’s motion, light that strikes the back of the compartment doesn’t have as far to go and strikes sooner than light that strikes the front of the compartment.

Spacetime

- Space and time are intimately linked together. Things exist in spacetime.
  - Each object, each person, each planet, each star, each galaxy exists in what physicists call "the spacetime continuum."
- One observer’s measurements of space and time differ from the measurements of another observer in some other realm of spacetime in such a way that each observer will always measure the same ratio of space and time for light: the greater the measured distance in space, the greater the measured interval of time.

\[
\frac{\text{Space}}{\text{Time}} = \frac{\text{Space}}{\text{Time}} = c
\]
Suppose that the observer standing on a planet sees a pair of lightning bolts simultaneously strike the front and rear ends of the compartment in a high-speed rocket ship. Will the lightning strikes be simultaneous to an observer in the middle of the compartment in the rocket ship?

A. Yes, they will be simultaneous.
B. No, they will be nonsimultaneous.
C. It depends upon how fast the ship is moving.
D. It depends upon how long the ship is.

Explanation: No; an observer in the middle of the compartment will see the lightning that hits the front end of the compartment before seeing the lightning that hits the rear end.
• Imagine that we are somehow able to observe a flash of light bouncing to and fro between a pair of parallel mirrors, like a ball bouncing to and fro between a floor and ceiling.

• If the distance between the mirrors is fixed, then the arrangement constitutes a light clock, because the back-and-forth trips of the flash take equal time intervals.

Light Clock on Spaceship

(a) An observer moving with the spaceship observes the light flash moving vertically between the mirrors of the light clock.

(b) An observer who sees the moving ship pass by observes the flash moving along a diagonal path.
Time Dilation

- Because the speed of light is the same in all reference frames (Einstein’s second postulate), the flash must travel for a correspondingly longer time between the mirrors in our frame than in the reference frame of the onboard observer.

\[ \frac{\text{Distance}}{\text{Time}} = \frac{\text{Distance}}{\text{Time}} = c\]

Time Dilation

- The relationship between the time \( t_0 \) (call it the proper time) in the frame of reference moving with the clock at speed \( v \) and the time \( t \) measured in another frame of reference (call it the relative time) is

\[ t = t_0 \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \]

\[ t = \gamma t_0 \]

\[ \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \] A factor \( \geq 1 \) that tells amount of relativistic effect
Time Dilation

- As the speed of a spaceship increases, the Lorentz factor increases as per the graph shown.
  - Clocks will tick slower and slower as the spaceship approaches the speed of light.

- When we see the rocket traveling at close to the maximum rate through space (the speed of light), we see its time practically standing still.

If you were moving in a spaceship at a high speed relative to Earth, would you notice a difference in your pulse rate or the pulse rate of people on Earth?

A. Yes, you would notice a difference in both pulse rates.
B. You would notice a difference in your pulse rate, but not the pulse rate of people on Earth.
C. You would notice a difference in the pulse rate of people on Earth, but not in your own pulse rate.
D. You would not notice a difference in either pulse rate.
If you were moving in a spaceship at a high speed relative to Earth, would you notice a difference in your pulse rate or the pulse rate of people on Earth?

A. Yes, you would notice a difference in both pulse rates.
B. You would notice a difference in your pulse rate, but not the pulse rate of people on Earth.
C. **You would notice a difference in the pulse rate of people on Earth, but not in your own pulse rate.**
D. You would not notice a difference in either pulse rate.

*Explanation:* There would be no relative speed between you and your pulse because the two share the same frame of reference. Therefore, you would notice no relativistic effects in your pulse. There would be, however, a relativistic effect between you and people back on Earth. You would find their pulse rate to be slower than normal.

---

Will observers A and B agree on measurements of time if A moves at half the speed of light relative to B?

A. Yes, they would agree completely.
B. No, they would disagree completely.
C. They would agree half of the time and disagree the other half of the time.
D. None of the above.
Will observers A and B agree on measurements of time if A moves at half the speed of light relative to B?

A. Yes, they would agree completely.
B. **No, they would disagree completely.**
C. They would agree half of the time and disagree the other half of the time.
D. None of the above.

*Explanation:* When A and B move relative to each other, each observes a slowing of time in the other’s frame of reference. So they do not agree on measurements of time.

---

Will observers A and B agree on measurements of time if both A and B move together at half the speed of light relative to Earth?

A. Yes, they would agree completely.
B. No, they would disagree completely.
C. They would agree half of the time and disagree the other half of the time.
D. None of the above.
<table>
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<td>CHECK YOUR ANSWER</td>
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*Explanation:* When they are moving in unison, they share the same frame of reference and agree on measurements of time. They see each other’s time as passing normally, and they each see events on Earth in the same slow motion.

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Does time dilation mean that time really passes more slowly in moving systems or only that it seems to pass more slowly?

A. **Time really passes more slowly in moving systems.**
B. Time only *seems* to pass more slowly in moving systems.
C. It depends upon how fast the system is moving.
D. It depends upon the direction in which the system is moving.

**Explanation:** The slowing of time in moving systems is not merely an illusion resulting from motion. Time really does pass more slowly in a moving system relative to one at relative rest.

---

**The Twin Trip**

- Identical twins, one an astronaut who takes a high-speed round-trip journey in the galaxy while the other stays home on Earth
  - When the traveling twin returns, he is younger than the stay-at-home twin.
  - How much younger depends on the relative speeds involved.
The Twin Trip

• Since motion is relative, why doesn't the effect work equally well the other way around? Why wouldn't the traveling twin return to find his stay-at-home twin younger than himself?
• The answer is that the situation is not symmetric. The Earth-bound twin stays in one reference frame the whole time, while the other accelerates to high speed, then later decelerates and reverses.

The Twin Trip
CHECK YOURSELF

Since motion is relative, can't we say as well that the spaceship is at rest and the Earth moves, in which case the twin on the spaceship ages more?

A. Yes.
B. No.
C. It depends on how fast the ship is moving.
D. It depends upon the direction in which the ship is moving.
Since motion is relative, can’t we say as well that the spaceship is at rest and the Earth moves, in which case the twin on the spaceship ages more?

A. Yes.
B. No.
C. It depends on how fast the ship is moving.
D. It depends upon the direction in which the ship is moving.

Explanation: The situation is not symmetrical, for one twin remains in a single reference frame in spacetime during the trip while the other makes a distinct change of reference frame, as evidenced by the acceleration in turning around.

Relativistic Addition of Velocities

- For everyday objects: \( V = v_1 + v_2 \)
  
  If you walk at 1 m/s on an airport ramp moving at 0.5 m/s, your total speed is 1.5 m/s

- Strictly speaking, the above rule is an approximation of the relativistic rule for adding velocities. We’ll not treat the long derivation but simply state the rule:

  \[
  V = \frac{v_1 + v_2}{\sqrt{1 + \frac{v_1 v_2}{c^2}}}
  \]

  If a spaceship moving at 0.5c relative to Earth shoots out a projectile moving at 0.5c relative to the spaceship, its speed relative to Earth is 0.8c.

- A consequence of this is that no material object can move as fast as or faster than the speed of light.
Length Contraction

- As objects move through spacetime, space as well as time changes.
- Space is contracted, making the objects look shorter when they move by us at relativistic speeds.

\[ L = \frac{L_0}{\gamma} \]

$L_0$ is length when stationary; $L$ is length when moving; $\gamma$ is Lorentz factor

Length Contraction

- Length contraction takes place only in the direction of travel.
  - If an object is traveling horizontally, no contraction takes place vertically.
Relativistic Momentum

• Relativistic momentum is:

\[ p = \gamma m_0 v \]

• \( m_0 \) is the “rest” mass. Classically \( p = m_0 v \)

• Subatomic particles are routinely pushed to nearly the speed of light. Classically, the particles behave as if their masses increase with speed.

• The increased momentum of a high-speed particle is evident in the increased “stiffness” of its trajectory.

Relativistic Momentum

• If the momentum of the electrons were equal to the Newtonian value \( mv \), the beam would follow the dashed line.

• But because the relativistic momentum \( \gamma mv \) is greater, the beam follows the “stiffer” trajectory shown by the solid line.
Mass, Energy and $E = mc^2$

• A piece of matter, even at rest and not interacting with anything else, has an “energy of being.” This is called its rest energy.

• *Einstein concluded that it takes energy to make mass and that energy is released if mass disappears.*

• The amount of energy $E$ is related to the amount of mass $m$ by the most celebrated equation of the 20th century:

$$E = mc^2$$

Mass, Energy and $E = mc^2$

• Saying that a power plant delivers 90 million megajoules of energy to its consumers is equivalent to saying that it delivers 1 gram of energy to its consumers, because mass and energy are equivalent.

• In 1 second, 4.5 million tons of mass are converted to radiant energy in the Sun through nuclear fusion reactions. The Sun is so massive, however, that in 1 million years only 1 ten-millionth of the Sun’s mass will have been converted to radiant energy.
Correspondence Principle

• States that any new theory or any new description of nature must agree with the old where the old gives correct results.

• If the equations of special relativity are valid, they must correspond to those of classical mechanics when speeds much less than the speed of light are considered.

• When speeds are very low, compared to the speed of light $v$ is much smaller than $c$, then

$$
\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1 - 0}} = 1
$$

Correspondence Principle

So:

• Relativistic time: 
  $$
t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{t_0}{\sqrt{1 - 0}} = t_0
$$

• Relativistic length: 
  $$
L = L_0 \sqrt{1 - \frac{v^2}{c^2}} = L_0 \sqrt{1 - 0} = L_0
$$

• Relativistic momentum: 
  $$
p = \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{mv}{\sqrt{1 - 0}} = mv
$$
Key Points of Lecture 41

Motion Is Relative
Postulates of the Special Theory of Relativity
Simultaneity
Spacetime
Time Dilation
The Twin Trip
Addition of Velocities
Length Contraction
Relativistic Momentum
Mass, Energy and $E = mc^2$
The Correspondence Principle

- Before Monday Dec. 13, read/skim *Hewitt* Chap. 35.
- Homework #27 due by 11:00 PM Friday Dec. 10
- Homework #28 due by 11:00 PM Monday Dec. 13