ASTR 400/700: Stellar Astrophysics

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Absolute Magnitude and Distance Modulus

$m - M$ is a measure of the distance to a star and is called the **distance modulus**.

$$m - M = 5 \log_{10}(d) - 5 = 5 \log_{10} \left( \frac{d}{10 \text{ pc}} \right).$$

The absolute magnitude of the Sun is $M = 4.83$. The luminosity of the Sun is $L = 3.846 \times 10^{26} \text{ W}$

$$M = M_{\text{Sun}} - 2.5 \log_{10} \left( \frac{L}{L_{\odot}} \right),$$

Note the $M$ includes only light in the visible band, so this is accurate only for stars with the same spectrum as the Sun.
The Quantization of Energy
Chapter 3.5
Definition of a black body

A black body is an ideal body which allows the whole of the incident radiation to pass into itself (without reflecting the energy) and absorbs within itself this whole incident radiation (without passing on the energy). This property is valid for radiation corresponding to all wavelengths and to all angles of incidence. Therefore, the black body is an ideal absorber of incident radiation.
Properties of Thermal Radiation

1. Hotter objects emit more light at all frequencies per unit area (Stefan-Boltzmann law). \( L = A\sigma T^4 \)

2. Hotter objects emit photons with a higher average energy (Wien's law). \( \lambda_{\text{max}} T = 0.002897755 \, \text{mK} \)
Rayleigh-Jeans Law

* It agrees with experimental measurements for long wavelengths.
* It predicts an energy output that diverges towards infinity as wavelengths grow smaller.
* The failure has become known as the ultraviolet catastrophe.

\[ B_\lambda(T) \approx \frac{2ckT}{\lambda^4}, \quad \text{(valid only if } \lambda \text{ is long)} \]
Planck Law

- We have two forms. As a function of wavelength.

\[ B_\lambda(T) = \frac{2hc^2/\lambda^5}{e^{hc/\lambda kT} - 1}. \]

And as a function of frequency

\[ B_\nu(T) = \frac{2h\nu^3/c^2}{e^{h\nu/kT} - 1}. \]

The Planck Law gives a distribution that peaks at a certain wavelength, the peak shifts to shorter wavelengths for higher temperatures, and the area under the curve grows rapidly with increasing temperature.
Motivation

• The black body is importance in thermal radiation theory and practice.
• The ideal black body notion is importance in studying thermal radiation and electromagnetic radiation transfer in all wavelength bands.
• The black body is used as a standard with which the absorption of real bodies is compared.
Planck’s Law and Astrophysics

Consider a model star consisting of a spherical blackbody of radius $R$ and temperature $T$. Assuming that each patch $dA$ emits isotropically over the outward hemisphere, the energy per second having wavelengths between $\lambda$ and $\lambda + d\lambda$ emitted by the star is:

$$L_{\lambda} \, d\lambda = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} \int_{A} B_\lambda \, d\lambda \, dA \, \cos \theta \, \sin \theta \, d\theta \, d\phi.$$ 

Integrating over a sphere results in:

$$L_{\lambda} \, d\lambda = 4\pi^2 R^2 B_\lambda \, d\lambda = \frac{8\pi^2 R^2 h c^2 / \lambda^5}{e^{hc/\lambda kT} - 1} \, d\lambda.$$ 

Monochromatic luminosity!
The Color Index
Chapter 3.6
Why aren't there any green stars?
Why aren't there any green stars?

Human spectral sensitivity to color

Three cone types (ρ, γ, β) correspond roughly to R, G, B.

[Graph showing human spectral sensitivity to color with peaks at different wavelengths for blue, cyan, green, and red.]
Color/Temperature Relation

What does the color of a celestial object tell us?

Betelguese (3500K)

Rigel (11000K)
What can we learn from a star’s color?

The color indicates the temperature of the surface of the star.

a. This star looks red
b. This star looks yellow-white
c. This star looks blue-white
**Bolometric Magnitude:** measured over all wavelengths.

**UBV wavelength filters:** The color of a star may be precisely determined by using filters that transmit light only through certain narrow wavelength bands:

- **U**, the star’s ultraviolet magnitude. Measured through filter centered at 365nm and effective bandwidth of 68nm.
- **B**, the star’s blue magnitude. Measured through filter centered at 440nm and effective bandwidth of 98nm.
- **V**, the star’s visual magnitude. Measured through filter centered at 550nm and effective bandwidth of 89nm.

U, B, and V are apparent magnitudes.
Optical Wavelength Bands

U: $\lambda_0 \approx 3650 \text{ Å}$
B: $\lambda_0 \approx 4400 \text{ Å}$
V: $\lambda_0 \approx 5500 \text{ Å}$
Power Density ($10^{12}$ W/m$^3$)

- Temperature = 1570 K
- Passband = 375–675 nm
- Peak occurs at 1846 nm
- Flux in passband = 0.06%

Wavelength (nm)
The Color Index

The color of a star is measured by comparing its brightness in different wavelength bands:

The blue (B) band and the visual (V) band.

We define B-band and V-band magnitudes just as we did before for total magnitudes.
The Color Index

We define the Color Index

\[ B - V \]

(i.e., B magnitude – V magnitude)

The \textit{bluer} a star appears, the \textit{smaller} the color index \( B - V \).

The \textit{hotter} a star is, the \textit{smaller} its color index \( B - V \).
Example:

For our sun:

Absolute V magnitude: 4.83

Absolute B magnitude: 5.51

\[ B - V = 0.68 \]

=> Color index:

From standard tables:

\[ B - V = 0.68 \quad \Rightarrow \quad T \approx 5800 \text{ K.} \]
Bolometric Correction

\[ U - B = M_U - M_B \]

\[ B - V = M_B - M_V. \]

Color is (kind of) independent of distance.
The difference between a star's bolometric magnitude and visual magnitude is its bolometric correction:

\[ BC = m_{bol} - V = M_{bol} - M_V. \]
Extinction and Reddening
Interstellar Reddening

One also needs to correct color indices for interstellar reddening. As the light propagates through interstellar dust, the blue light is scattered preferentially making objects appear to be redder than they actually are...
Interstellar Extinction
Wavelength-Dependent Extinction

![Graph showing wavelength-dependent extinction](image)

- **BD+56 524**: $R = 2.75$
- **HD 48099**: $R = 3.52$
- **Herschel 36**: $R = 5.30$

Where $A_\lambda / A_V$ is plotted against $1/\lambda$ (um$^{-1}$).