The Classification of Binary Stars
Chapter 7.1
Flowchart of Key Stellar Parameters

- Parallax ($p$)
  - $d = \frac{1}{p}$
  - Distance ($d$)
- Apparent brightness ($b$)
  - $L = 4\pi d^2 b$
  - Luminosity ($L$)
- Spectrum
  - Spectral type
  - Surface temperature ($T$)
  - Chemical composition
- $L = 4\pi R^2 \sigma T^4$
- Radius ($R$)
Stellar Mass

- Fuel burning rate
- Lifetime $10^{10}$ yr $(M/M_{\text{Sun}})^{-2.8}$
- Luminosity $L \propto M^{3.8}$
- Impossible to measure for isolated stars
How do we measure stellar masses?

Binary Star Orbits

Two stars held in orbit around each other by their mutual gravitational attraction. Each of the two stars in a binary system moves in an elliptical orbit about the center of mass of the system. Orbit of a binary star system depends on the component masses.
The center of mass of the binary star system is nearer to the more massive star.
About half of all stars are in binary systems.
Most Sub-Arcsecond Companions of Kepler Exoplanet Candidate Host Stars are Gravitationally Bound

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ABSTRACT

Using the known detection limits for high-resolution imaging observations and the statistical properties of true binary and line-of-sight companions, we estimate the binary fraction of Kepler exoplanet host stars. Our speckle imaging programs at the WIYN 3.5-m and Gemini North 8.1-m telescopes have observed over 600 Kepler objects of interest (KOIs) and detected 49 stellar companions within ~1 arcsecond. Assuming binary stars follow a log-normal period distribution for an effective temperature range of 3,000 to 10,000 K, then the model predicts that the vast majority of detected sub-arcsecond companions are long period ($P > 50$ years), gravitationally bound companions. In comparing the model predictions to the number of real detections in both observational programs, we conclude that the overall binary fraction of host stars is similar to the 40-50% rate observed for field stars.
Astrometric Binary

Unseen companion

Center of mass

Visual member

Right ascension

Declination

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Astrometric Binary: Sirius AB

- Sirius A:
  - nearby luminous B star
  - brightest star in the sky
- $\sim 1 \, M_{\text{Sun}}$ white dwarf companion first inferred from its large astrometric effect on primary
- now also a visual binary
Astrometric Binary: GJ 802AB

unseen brown dwarf companion

$a > 0.5 - 2\text{AU}$

(Pravdo et al. 2005)
**Eclipsing Binary**

We can measure periodic eclipses.

We see light from both stars A and B.

We see light from all of B, some of A.

We see light from both A and B.

We see light only from A (B is hidden).

We can measure periodic eclipses.
Eclipsing Binary

Phased lightcurve for YY Sagittarii.
Algol – Eclipsing Binary

Algol (the demon star) is in the constellation of Perseus.

• Algol A: main sequence star, more massive.
• Algol B: subgiant, less massive.

⇒ The Algol Paradox:

Why has the more massive Algol A evolved slower than the less massive Algol B?
Spectroscopic Binary

- Sometimes only the spectrum from one star is seen, the other star is too dim.
- Sometimes two sets of spectra can be seen at the same time.
- Sometimes more than two sets of spectra can be seen.
  - *Mizar* is a visual binary system in the constellation of *Big Dipper*.
  - Each ‘star’ in the visual binary system is also a spectroscopic binary!

[Diagram of Mizar showing the visual and spectroscopic binary systems.]
Spectroscopic Binary

• double-lined (SB2)
  - spectra of both stars visible

• single-lined (SB1)
  - only spectrum of brighter star visible
Mass Determination

Chapter 7.2
Need two out of three observables to measure mass:

1. Orbital period \((P)\)
2. Orbital separation \((a\text{ or } r = \text{radius})\)
3. Orbital velocity \((v)\)

For circular orbits, \(v = \frac{2\pi r}{P}\)
Centre of mass of binary system

Distance is related to the angular semi-major axes.

\[ \alpha_1 = \frac{a_1}{d} \quad \text{and} \quad \alpha_2 = \frac{a_2}{d}, \]

Using Kepler's third law we determine the sum of the masses.

\[ m_1 + m_2 = \frac{4\pi^2}{G} \left( \alpha d \right)^3 = \frac{4\pi^2}{G} \left( \frac{d}{\cos i} \right)^3 \tilde{\alpha}^3 \left( \frac{1}{P^2} \right), \]
Radial Velocity vs. Time for Double-lined SB in a Circular Orbit

(a) Schematic diagram showing a double-lined spectroscopic binary (SB) system in a circular orbit. The radial velocities $v_1$ and $v_2$ are indicated, along with the common center of mass $v_{cm}$.

(b) Graph showing the radial velocity $v_1$ and $v_2$ as a function of the orbital phase $t/P$. The graph displays a periodic oscillation with a maximum amplitude of approximately 120 km s$^{-1}$. The center of mass velocity $v_{cm}$ is indicated.
Radial Velocity vs. Time for Double-lined SB in Elliptical Orbit \((e = 0.4)\)
Luminosity-Mass Relation for Stars with Well-determined Orbits

- Detached main-sequence systems, B6 to M
- Visual binaries
- Detached OB systems
- Resolved spectroscopic binaries

(Popper 1980)
Eclipsing Binaries

Chapter 7.3
Light curves of eclipsing binaries provide detailed information about the two stars

• An eclipsing binary is a system whose orbits are viewed nearly edge-on from the Earth, so that one star periodically eclipses the other

• Detailed information about the stars in an eclipsing binary can be obtained from a study of the binary’s radial velocity curve and its light curve
(a) Partial eclipse

(b) Total eclipse

(c) Tidal distortion
Totally Eclipsing Binaries

- $t_a$ – start of secondary ingress
- $t_b$ – end of secondary ingress
- $t_c$ – start of secondary egress
- $t_d$ – end of secondary egress
Other Uses for Totally Eclipsing Binary Systems: Radii and $T_{\text{eff}}$’s

- Duration of eclipses and shape of light curve can be used to determine radii of stars:

  \[
  R_s = \frac{v_1 + v_2}{2} (t_2 - t_1) \\
  R_\ell = \frac{v_1 + v_2}{2} (t_3 - t_1)
  \]

  \(t_1\) – start of secondary ingress \(t_2\) – end of secondary ingress \(t_3\) – start of secondary egress

Relative depth of primary (deepest) and secondary brightness minima of eclipses can be used to determine the ratio of effective temperatures of the stars:

\[
\frac{F_0 - F_{\text{primary}}}{F_0 - F_{\text{secondary}}} = \left(\frac{T_{\text{e,s}}}{T_{\text{e,\ell}}}\right)^4.
\]
Examples of radial velocity data

**51 Pegasi**

- Mass = $0.46 \, M_{\text{Jup}} \, / \sin i$
- $P = 4.230 \, \text{day}$
- $K = 55.9 \, \text{m s}^{-1}$
- $e = 0.00$

**RMS** = $5.23 \, \text{m s}^{-1}$

Lick Obs.
Example of a planet with an eccentric orbit: $e=0.67$. 

**HD 89744**

- Mass: $7.56 \, M_{\text{Jup}} / \sin i$
- $P = 256.3 \, \text{day}$
- $K = 262.9 \, \text{m s}^{-1}$
- $e = 0.67$

**RMS = 13.3 \, \text{m s}^{-1}$**

**Lick Obs.**
A Triple-Planet System Orbiting Upsilon Andromedae
Kepler-5 b
Transit duration

An observing program is based on the duration and frequency of expected transits.

The frequency of transits is given by one over the orbital period:

$$P = \sqrt{\frac{4\pi^2 a^3}{GM_*}}$$

Transit duration is the fraction of the orbital period during which part of the planet eclipses the stellar disk.

Duration $\equiv t_T = 2P \arcsin \left( \frac{\sqrt{(R_* + R_p)^2 - a^2 \cos^2 i}}{a} \right)$
For $a >> R_* >> R_p$ this becomes:

$$t_T = \frac{P}{\pi} \sqrt{\left(\frac{R_*}{a}\right)^2 - \cos^2 i} \leq \frac{PR_*}{\pi a}$$
What can we measure from the light curve?

(1) Depth of transit = fraction of stellar light blocked

\[
\frac{\Delta F}{F_0} = \left( \frac{R_p}{R_*} \right)^2
\]

Measure of planetary radius
In practice, isolated planets with masses between ~ 0.3 \( m_J \) and 10 \( m_J \), where \( m_J \) is the mass of Jupiter, have almost the same radii (i.e. a flat mass-radius relation).

-> Giant extrasolar planets transiting solar-type stars produce transits with a depth of around 1%

Close-in planets are strongly irradiated, so their radii can be (detectably) larger

(2) Duration of transit plus duration of ingress, gives measure of the orbital radius and inclination

(3) Bottom of light curve is not actually flat, providing a measure of stellar limb-darkening

(4) Deviations from profile expected from a perfectly opaque disc could provide evidence for satellites, rings etc

(5) Since inclination is approximately known, can constrain mass estimates from radial velocity measurements