An Electronic Measurement of the Boltzmann's Constant Using I-V Characteristics of a Silicon 2N3904 *Diode*

Shawn Kann

Department of Physics and Astronomy, San Francisco State University, San Francisco, California 94132

Abstract: Previous studies have shown that Boltzmann's constant k can be determined using *Diode*. In this study we investigated the Boltzmann's constant by measuring the current-voltage characteristic of a 2N3904 Diode. The experimentally measured value of $k = 1.35 \times 10^{-23}$ JK^{-1} is consistent with the current national institute of science and technology (NIST) value.

INTRODUCTION/THEORY

A p-n junction diode is a material that has a potential difference between the p-type and the n-type junction regions in semiconductors. Within the junction region there exists a contact potential V_0 due to electrons diffusion across *n*-type region to *p*-type region. An electron entering in this potential well acquires potential energy and depletes its kinetic energy (figure 1).[1] Once the junction or depletion region is established, the displacement of electrons from the n-type region to the *p*-type region occurs only if electrons have sufficient energy to overcome the contact potential energy barrier eV_0.[1] According to statistical mechanics, the number of electrons displaced across the junction is governed by the Boltzmann distribution law which states that the probability of an electron having energy eV_0 is dictated by:

$$i_d = ae^{-eV_0/kT} \qquad (1)$$

where i_d is the displaced current, a is a proportionality constant, T is the *temperature* and k is *Boltzmann's constant*.[2]

The displaced current generates an electric field $\mathbf{E_d}$ in the depletion region and if forward biased voltage is applied so that *p*-type region is made more positive with relation to *n*-type region then the electric field generated by the external biased voltage V is $\mathbf{E_D}$ that points from *p*-type region to *n*-type region (figure 1).[1] The induced voltage V alters the current by:

$$i_d = ae^{-e(V_0-V)/(kT)}$$

The total current then is described by Shockley diode equation:

$$I = ae^{-e(V_0-V)/(kT)} - ae^{-eV_0/kT}$$

$$I = I_0(e^{eV/kT} - 1) \qquad (2)$$

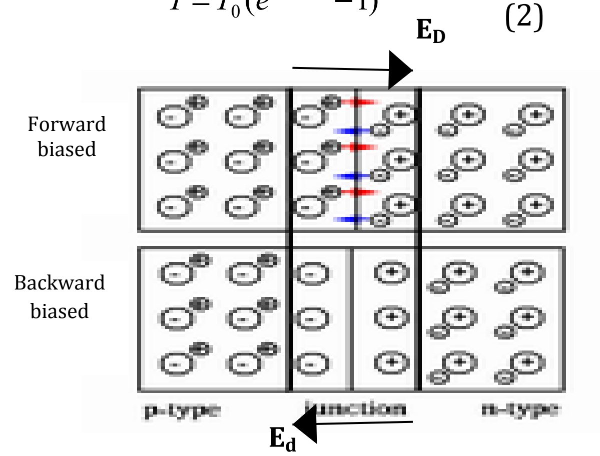

Figure 1. A p-n junction diode.

A diode is composed of a *p*-type region known as the *cathode* and an *n*-type region known as the *anode*.[3] Diode has the ability to conduct electrons from the *anode* to the *cathode* and prevent electron flow from anode to cathode.

EXPERIMENTAL METHODS

A *2N3904 Transistor* and *LF356* operational amplifier were chosen for this experiment. The Op Amp was used to covert current to voltage and was maintained at ± 15 V supply. The circuit used in this experiment is shown in Fig. 2 and was built on *Global Specialties Protoboard-503 Circuit Board*. The voltage was measured using two *972A digital Hewlet Packard* multimeters, one to measure the potential at point A (relative to ground) and the other to measure the potential at point B within the circuit.

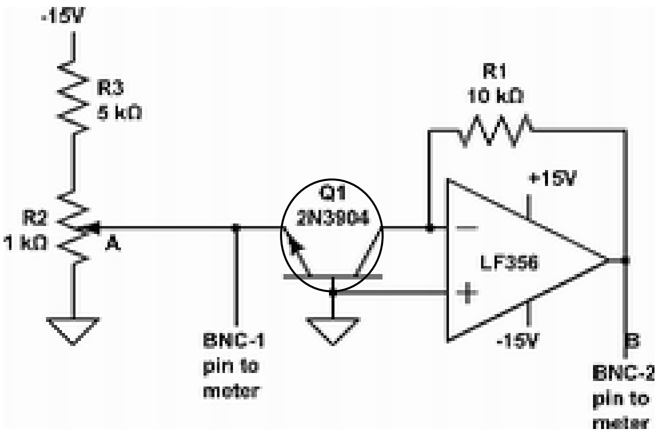

Figure 2: Circuit used to determine Boltzmann's constant.

The 1K potentiometer on PB-503 was calibrated to provide V_A reading of 0V and a very small voltage at V_B. Next, the 1K potentiometer was adjusted until V_B was 1V and the corresponding V_A was about -0.6V. The data acquisition at V_A and V_B was continued while adjusting the potentiometer to increment V_B by about 0.5V until V_B was about 12V. The temperature of the diode was assumed to be room temperature.

The following table shows the results obtained by following this method:

Table 1: Measured values of V_A and V_B. The uncertainty in V_A was found to be ± 0.001 V.

V_B (volts)	V_A (volts)	V_B (volts)	V_A (volts)
1.02	-0.616	7.02	-0.664
1.50	-0.626	7.52	-0.666
2.03	-0.633	8.01	-0.667
2.49	-0.639	8.51	-0.669
3.00	-0.643	9.01	-0.670
3.50	-0.647	9.51	-0.671
4.02	-0.650	10.02	-0.673
4.52	-0.653	10.52	-0.674
5.03	-0.656	11.06	-0.675
5.49	-0.658	11.51	-0.676
6.00	-0.660	12.05	-0.678
6.53	-0.662	12.56	-0.679

RESULTS

Since $e^{eV/kT}$ increases exponentially with the variable V_A we can ignore the second term in equation 2, giving

$$I = I_0 e^{eV/kT} \qquad (3)$$

Taking the natural log of the equation gives,

$$\ln(I) = \ln(I_0) + (eV_A / kT)$$

By plotting a graph (Figure 3) of the natural log of the current passing through the diode versus the voltage across and using a least-square-fit plot in MATLAB with the plot1 script we can determine a close fitting slope in the form:

$$y = mx + b \Rightarrow$$

$$\ln(I) = (e/kT)V_A + \ln(I_0) \qquad (4)$$

where V_A is variable and the slope is given by e/kT. Using the slope we can determine the Boltzmann's constant. The graph in (Figure 3) has a slope with a value of about 40.22 (the units are discovered later when solving for the Boltzmann's constant). The temperature taken in the lab was approximately 293 kelvin and an electron has a charge with an absolute value of 1.60 x 10^{-19} Coulomb.

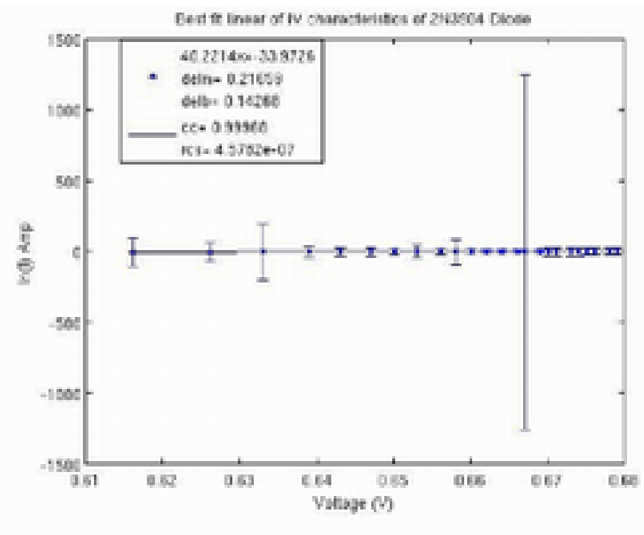

Figure 3: I-V characteristic of *npn* 2N3904 diode.

ANALYSIS

The experimental value of Boltzmann's constant is obtained using equation 4 where using the value of the slope yields the experimental value of Boltzmann's constant,

$$k = \frac{1.60 \times 10^{-19} C}{(40.22)(293 K)} = 1.35 \times 10^{-23} J/K$$

which is correct in order of magnitude with the current accepted value of 1.38 x 10^{-23} as given by NIST. The units for the Boltzmann's constant were determined by doing dimensional analysis using Equation 2.

$$I = I_0 (e^{eV/kT} - 1)$$

it is trivial that the following relation holds

$$1 = \frac{eV}{kT} = \frac{CV_A}{kK}$$

solving for *k* yields

$$[k] = \frac{J}{K}$$

The theoratical value of the Boltzmann's constant according to NIST is 1.38 x 10^{-23} J/K and the experimental value of the Boltzmann's constant is 1.35 x 10^{-23} J/K. Comparing the results with accepted value the descrpency is,

$$\frac{1.38 \times 10^{-23} J/K - 1.35 \times 10^{-23} J/K}{1.35 \times 10^{-23} J/K} \times 100 = 2.22\%$$

CONCLUSIONS

This study has demonstrated that the Shockley Diode equation is a suitable model of the correlation between the current through a diode and the voltage across the diode. The graph in (figure 3.) shows a strong linear relationship as predicted by the Shockley Diode equation.

Acknowledgement

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References

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