Friction on an incline

Static Friction $f_s \leq \mu_s N$ is only as large as it needs to be to maintain equilibrium, up to the maximum value $f_{\text{max}} = \mu_s N$. As the angle of an incline increases, the static friction force on a mass at rest on the incline will also increase until its maximum is reached. For steeper angles, the mass will slide and kinetic friction takes over.

\[ f_s = \mu_s N \leq mg \cos \theta \]

For $\theta = \theta_c$, the mass will slide at constant speed, with kinetic friction $f_k = \mu_k N$ balancing $mg \sin \theta$. For $\theta > \theta_c$, the mass will slide at constant speed, with kinetic friction $f_k = \mu_k N$ balancing $mg \sin \theta$.

\[ f_k = \mu_k N \]

\[ \sum F_y = 0 \]

\[ N - mg = 0 \]

\[ N = mg \]

\[ \sum F_x = 0 \]

\[ mg \sin \theta - f_s = 0 \]

\[ f_s = mg \sin \theta \]

\[ f_s < \mu_s N \]

\[ \theta_c = \tan^{-1} \mu_s \]

\[ N = mg \cos \theta \]

This is the steepest angle at which we don't slide, or approximately the shallowest angle at which we do slide.

\[ mg \sin \theta = \mu N \]

\[ \mu N = mg \sin \theta \]

\[ \mu \tan \theta = \mu_k \]

\[ a = g (\sin \theta - \mu_k \cos \theta) \]