The work done by a force $\vec{F}$ on an object as it travels along a path from $\vec{F}_0$ to $\vec{F}_f$ is

$$ W = \int_{F_0}^{F_f} \vec{F} \cdot d\vec{r} $$
units: Joules
$$ 1 J = 1 N m $$

In general, the work depends on the path taken from $\vec{F}_0$ to $\vec{F}_f$. The work done by gravity, or a spring, is path independent.

The Dot Product or Scalar Product

The dot product of two vectors is defined as $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y$ (or $A_x B_x + A_y B_y + A_z B_z$ for 3D).

or alternatively

$$ \vec{A} \cdot \vec{B} = AB \cos \theta $$ gives the same result.

where $\theta$ is the angle between $\vec{A}$ and $\vec{B}$.

\[
\begin{align*}
\vec{A} \cdot \vec{B} &= AB \cos \theta > 0 \\
\vec{A} \cdot \vec{B} &= 0 \\
\vec{A} \cdot \vec{B} \cos \theta < 0
\end{align*}
\]

Kinetic Energy

The total work, or net work done on an object by all forces acting on it is equal to the change in kinetic energy.

$$ W_{net} = \int (\sum F) \cdot d\vec{r} = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = \Delta K $$

Kinetic Energy $K = \frac{1}{2} m v^2$ units: Joules.

$\Delta K$ is called the **work-energy theorem** when the total work done on an object is $>0$, it speeds up. When $< 0$, the object slows down.

What is the work done by gravity on a ball of mass $m$ thrown vertically to height $h$?

What is the work done by gravity as the ball falls from height $h$ back to $y=0$?

How much work is needed to compress a spring of spring constant $k$ a distance $x$?

What is the work done by the Earth on the Moon during one complete lunar orbit?

(a) $< 0$ (b) $0$ (c) $> 0$

What is the work done by gravity on a ball of mass $m$ fired with speed $v_0$ at an angle $\theta$ above horizontal once it reaches its maximum height, and once it returns to the ground?

A steady wind exerts a force $F$ at an angle $\theta$ south of east on a plane that follows a trajectory east at speed $v$ for time $t$.

What is the work done by the wind?

(a) $0^\circ$ (b) $90^\circ$ (c) $180^\circ$ (d) $270^\circ$

What is the minimum stopping distance of a truck with initial speed $v_0$ if the coefficient of friction between tires and road is $\mu$?

What is the final speed of a cart released from rest once it has moved a distance $d$ down an incline of angle $\theta$?
Simple Machines / Mechanical Advantage

When you push a mass up a frictionless incline at constant speed, the force of your push plus the normal force of the incline point up, to balance the weight down.

\[ F_{push} \rightarrow V_{const.} \quad \text{FBD:} \quad F_{push} - mg = 0 \]

Consider the push force to be the input force and the combined normal force and push to be the output force for the incline as a simple machine.

**Power**

Power is the rate at which energy is given to or taken away from an object, or the rate at which a force does work.

\[ P = \frac{dW}{dt} = F \cdot V \]

\[ P_{total} = \frac{dW_{net}}{dt} = (\Sigma F) \cdot V = \frac{dK}{dt} \]

The power needed to push a mass \( m \) against friction \( \mu_k \) at constant speed \( V \) is \( P_{push} = \mu_k mg \cdot V \), \( P_{friction} = -\mu_k mg \cdot V \), \( P_{total} = 0 \).

**Potential Energy**

A force is conservative if the work done on any object by the force depends only on the object’s initial and final positions, and not on the path taken. Conservative forces are path-independent.

Free Fall / Frictionless: Any frictionless path, \( V_f = 0, V_i = h \), same final speed.

\[ \Delta K = mgh \Rightarrow V_f = \sqrt{2gh} \quad \text{for} \quad V_i = 0 \]

The work done by a conservative force defines a change in potential energy \( \Delta U = -W_c \).