Gravitational Potential Energy near Earth’s surface

Gravitational force (weight) always points down, \( F = -mg \)

\[ \Delta U = -W_c = - \int_{r_0}^{r_f} (-mg) \cdot \frac{dr}{r} \]

\[ = mg \cdot \Delta r = mg \Delta y \]

where \( y \)-axis is vertical, and \( y = 0 \) is arbitrary, wherever convenient.

\( U = mgh \) is the P.E. for a mass at elevation \( h \) above \( y = 0 \).

Elastic Potential Energy

A spring stores potential energy when compressed or stretched, and releases it when it returns to equilibrium. For a mass on a spring moving in the \( x \)-direction:

\[ \Delta U = - \int_{x_0}^{x_f} (-kx) \cdot dx = \frac{1}{2} k (\Delta x)^2 \]

\[ U = \frac{1}{2} kx^2 \] for \( x = 0 \) at equilibrium.

Gravitational Potential Energy of satellites and spaceships.

\[ \Delta U = - \int_{r_0}^{r_f} \frac{GMm}{r^2} \cdot dr \]

\[ = -GMm \left( \frac{1}{r_f} - \frac{1}{r_0} \right) \]

Choosing \( U = 0 \) at \( r_0 \rightarrow \infty \)

\[ U = -GMm \]

where \( r \) is the distance from the center of mass of \( M \).

Kepler’s 3rd Law, \( T^2 \propto a^3 \)

Let’s us compare the periods or semi-major axes of satellites in different orbits. Energy Conservation tells us compare speeds at different points on the same orbit.

Let \( r_a \) = aphelion, \( v_a \) the speed at \( r_a \), \( r_p \) = perihelion, and \( v_p \) the speed at \( r_p \).

\[ \frac{1}{2} m v_a^2 = \frac{GMm}{r_a} \]

\[ \frac{1}{2} m v_p^2 - G \frac{mM}{r_p} \]

What minimum speed \( v_e \) is needed to escape the Earth’s gravity starting from its surface?