more examples with Moment of Inertia

Parallel Axis Theorem

\[ I_{\parallel} = I_{cm} + Mx^2 \]

distance from \( cm \) to center of rotation

Tell's us how much an object resists rotational acceleration about an axis displaced from the \( cm \).

For \( 2 \) objects rotating about a common axis

\[ I_{\text{tot}} = I_1 + I_2 \]

e.g. thick ring

\[ I_{\text{ring}} = I_{\text{disk 2}} - I_{\text{disk 1}} \]

\[ = \frac{1}{2}m_2R_2^2 - \frac{1}{2}m_1R_1^2 \]

\[ = \frac{1}{2} \pi \left( R_2^4 - R_1^4 \right) = \frac{1}{2} \pi \left( R_2^2 - R_1^2 \right) \left( R_2^2 + R_1^2 \right) = \frac{1}{2} m_{\text{ring}} \left( R_2^2 + R_1^2 \right) \]

Perpendicular Axis Theorem

\[ I_z = I_x + I_y \]

Find the moment of inertia of a disk about an axis in the plane of the disk through its \( cm \).

A uniform rod rotating about one end.

\[ I = \frac{1}{12}ML^2 + M \left( \frac{L}{2} \right)^2 \]

\[ = \frac{1}{3}ML^2 \]

\[ I = \int r^2 dm = \int_0^L r^2 \frac{M}{L} dr \]

\[ = \frac{1}{3}L \int_0^L \frac{M}{L} \left( L - r \right) dr = \frac{1}{3}ML^2 \]

A disk rotating about a point on its edge.

\[ I = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2 \]

\[ I_{\text{ring}} + I_{\text{disk 1}} = I_{\text{disk 2}} \]

\[ m_1 = \sigma \pi R_1^2, \quad m_2 = \sigma \pi R_2^2, \quad m_{\text{ring}} = m_2 - m_1 \]

A disk rotating about a point on its edge.

\[ I_x = I_y = \frac{1}{4}MR^2 \]

\[ \frac{1}{2}MR^2 = I_x + I_y \]

\[ = 2I_x \]

\[ I_x = I_y \text{ by symmetry} \]

\[ I_z = I_{\text{ring}} \text{ for disk} \]

about the axis \( \perp \) to the plane of the disk.