Homework 3 Due 1PM March 22

While I may have consulted with other students in the class regarding this homework, the solutions presented here are my own work. I understand that to get full credit, I have to show all the steps necessary to arrive at the answer, and unless it is obvious, explain my reasoning using diagrams and/or complete sentences.

Name ______________________________ Signature: ______________________________

1. (5 points) Liddle 7.1

2. (25 points) Mathematica problem: Consider a universe without dark energy (no cosmological constant). It has current matter density $\Omega_{M0}$ and radiation density $\Omega_{R0}$, each of which may take on any positive real value.

   (a) (10 points) Use Mathematica to solve the Friedman equation to get the age of this universe as a function of $\Omega_{M0}$ and $\Omega_{R0}$.

   (b) (5 points) Use the Plot3D command to make a 3D plot of the age for $\Omega_{M0} < 2$ and $\Omega_{R0} < 2$. Use the ContourPlot command to view this as a set of contours.

   (c) (5 points) Find the present-day age of a pure radiation (no matter, no $\Lambda$) universe in terms of the current radiation density.

   (d) (5 points) Find the age under the flat universe constraint $\Omega_{R} + \Omega_{M} = 1$. Under this constrain, compare the age of an $\Omega_{R} = 1$ universe to the age of an $\Omega_{M} = 1$ universe.

3. (15 points) Liddle 7.2. Use the fluid equation to argue that Einstein’s static universe is unstable.

4. (5 points) Liddle 7.3.

5. (20 points) Liddle 7.5.

6. (10 points) Liddle 8.3.

7. (15 points) Imagine a homogeneous and isotropic universe filled with a fluid which has equation of state $p = C\rho^3$, where $\rho$ is always positive and $C$ is a real constant.

   (a) (15 points) Use the fluid equation to prove that there is no big bang if $C > 0$.

   (b) (10 points) Show that if $C < 0$ then this universe has a big bang. Derive the condition under which the density is always decreasing as the scale factor $a$ increases. Does this fluid resemble anything we might have discussed in class?