CMB anisotropies

observations constrain the geometry of the universe, suggesting that the curvature constant $k = 0$. 
\[ \frac{\Delta T}{T}(\theta, \phi) = \frac{T(\theta, \phi) - \bar{T}}{\bar{T}}. \quad (A5.4) \]

\[ \frac{\Delta T}{T}(\theta, \phi) = \sum_{\ell=1}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{m}^{\ell}(\theta, \phi). \quad (A5.5) \]

The coefficients \( a_{\ell m} \) tell us the size of the irregularities on different scales. As with the galaxy distribution, to compare with theory we are interested only in the statistical properties of these coefficients, quantified by the **radiation angular power spectrum**, now known universally by the notation \( C_{\ell} \) and defined by

\[ C_{\ell} = \langle |a_{\ell m}|^2 \rangle. \quad (A5.6) \]
The most prominent feature in the cosmic microwave background is the $\ell = 1$ perturbation, known as the dipole. It corresponds to a pattern which is hot in one direction and cold in the opposite direction, with a smooth transition between them. It is believed to be due to the motion of the Earth relative to the microwave background, with the dipole simply due to the Doppler effect. Averaging over a year, its maximum value is $\Delta T/T = 1.23 \times 10^{-3}$, corresponding to the Sun having a velocity of $370\, \text{km}\,\text{s}^{-1}$; taking into account the Sun’s revolution around the galaxy this is consistent with the typical peculiar velocities observed for nearby galaxies. While interesting, this observation is not telling us about properties intrinsic to the microwave background, and so usually the dipole is studied separately and $\ell = 2$ is the smallest value considered. Maps of the cosmic microwave background are always shown with the dipole already removed.
Because the anisotropies in the cosmic microwave background represent small departures from homogeneity, it is possible to calculate them accurately, though this requires a sophisticated numerical computation which includes many physical processes, such as gravitational attraction and the interaction of radiation with electrons. Before the microwave background is released, the photons are interacting strongly with the electrons, providing a pressure which opposes gravitational collapse. At that time, therefore, the cosmic fluid is undergoing oscillations, alternating between compression and rarefaction under the combined influence of gravitation and pressure.

At some stage in this process, the Universe cools sufficiently to release the microwave background. This removes the pressure support from the atoms, which are now able to collapse gravitationally to form galaxies and stars. But the photons are already on their way towards us, carrying a snapshot of the complicated fluid motions taking place at a redshift of around one thousand.
Figure A5.3  A typical prediction of cosmic microwave anisotropies, in this case for the Standard Cosmological Model. The predicted curve is calculated to better than one percent accuracy.
the combination $\ell(\ell + 1)C_\ell/2\pi$ is plotted, and on the very largest scales, corresponding to small $\ell$ (for example COBE probed only $\ell \leq 15$), this has a non-zero value, with $\ell(\ell + 1)C_\ell$ roughly constant across the COBE range. This region is known as the Sachs–Wolfe plateau, and is caused by variations in the gravitational potential between regions. Moving to smaller scales (larger $\ell$) we see a broad peak at $\ell \approx 200$ followed by a series of peaks and troughs which represent the complicated fluid motions that were taking place when the microwave background was released. Bearing in mind our rule-of-thumb for relating $\ell$ to angles, $\theta \sim 180^\circ/\ell$, the first broad peak at $\ell \approx 200$ corresponds to an angular scale of about one degree, indicating that maps of the microwave background are predicted to have particularly strong features of that angular size.

Had I chosen a different cosmological model, the qualitative pattern of peaks and troughs would have been the same, but the detailed structure would change. Sufficiently accurate measurements of those structures can therefore rule out cosmological models.
After the discovery of the anisotropies by the COBE satellite, which only probed the largest angular scales, most observers turned their attention to probing the structure predicted on smaller angular scales, requiring higher-resolution experiments. While many experiments contributed towards this goal, it is widely recognized that the landmark step was made in April 2000 with the announcement of results from the Boomerang experiment. This was an ingenious experiment carried around Antarctica on a high-altitude balloon by wind currents, in order to maximize observation time (around fourteen days) while minimizing contamination from the atmosphere. This experiment clearly picked out the first broad peak, locating it at $\ell \approx 200$, the significance of which will be explored in the following subsection. It was rapidly followed by independent confirmation from another balloon experiment, called Maxima. Subsequently, more detailed analysis of these observations, along with results from new experiments, began to pick out the peak structures at larger $\ell$. 
Experiments carried out on Earth or on high-altitude balloons can have high precision, but their statistical power is limited by being able to survey only small areas of sky. In February 2003, spectacular results were announced by the WMAP satellite project. The true successor to COBE, this mission combined sensitive detectors with full sky coverage at a resolution approaching 10 arcminutes. The measurement of the radiation angular power spectrum based on their first year of observations is shown in Figure A5.4. A comparison with Figure A5.3 (noting the different $\ell$-axis scaling) shows that these observations agree extremely well with expectations.
Figure A5.4 The radiation angular power spectrum as measured by the WMAP satellite, shown as the black dots. The solid line shows a theoretical prediction from their best-fit cosmological model, which fits the data extremely well. They define $C_\ell$ using the multipoles of $\Delta T$ itself rather than $\Delta T/T$, so their scale is $T_0^2 = (2.725 \text{ K})^2$ times that of Figure A5.3.
While high-precision microwave background observations can constrain many cosmological parameters, one of the most important is that they give a direct indication of the geometry of the Universe, which can be read off from the location of the first peak in the angular power spectrum. While there is much complicated physics taking place around the time of formation of the microwave background, there is only one important characteristic scale, which is the Hubble time $H^{-1}$ at that redshift. The peak structure comes from oscillations, and so the first peak, being on the largest scale, must correspond to perturbations which have just had time to undergo one oscillation. The Hubble time is an estimate of the age of the Universe at that time, and the Hubble length $cH^{-1}$ estimates the physical size of a perturbation which oscillates on that timescale.²
The WMAP satellite results indicate that

$$\Omega_0 + \Omega_\Lambda = 1.02 \pm 0.02,$$

placing the Universe within a few percent of spatial flatness and completely ruling out significantly non-flat Universes. While this conclusion does depend on the validity of the assumptions made in the theoretical calculation of the $C_\ell$, those predictions are borne out so well by the observations that discussion of non-flat Universes has become very rare. Fig-
Figure A2.4 Angular diameter distance is a measure of how large objects appear to be. As with the luminosity distance, it is defined as the distance that an object of known physical extent appears to be under the assumption of Euclidean geometry. If we take the object to lie perpendicular to the line of sight and to have physical extent, the angular diameter distance is therefore

\[ \text{Angular diameter distance} = \frac{\text{physical extent}}{\text{angular size}} \]

Expands forever

Reco\apses

Accelerating
Decelerating

No Big Bang

Closed
Open

Expands to Infinity

Accelerating
Decelerating

Recollapses

Boomerang

High-Z SN Search Team

Supernova Cosmology Project

- 99.7%
- 95.4%
- 68.3%

\( \Omega_0 \)

\( \Omega_\Lambda \)
Gravitational instability is a powerful idea which lets us understand how structures in the Universe evolve. However, it does not let us address a more fundamental question — what is the origin of structure? Gravitational instability is excellent for taking initially small irregularities and amplifying them, but it needs the initial irregularities to act upon. Where might they come from?

The origin of structure takes us back into the realm of the very early Universe, because it appears that none of the established physics we know about is capable of making perturbations. However we do know of a mechanism that can. I introduced inflation in Chapter 13 following the historical motivation of the flatness and horizon problems. But in fact the best reason for believing in inflation (and certainly the best hope for testing the idea observationally) wasn’t appreciated until a year after Guth’s paper, and is that inflation can generate irregularities capable of initiating structure formation.
The mechanism is a remarkable one, being quantum mechanical in origin. Heisenberg's famous Uncertainty Principle tells us that even apparently empty space is a seething mass of quantum fluctuations, with particles continually popping in and out of existence. Normally we don't notice this as the time and length scales are so small, but during a period of inflation the Universe is expanding so rapidly that any fluctuations get caught up in the expansion and stretched. While one set of fluctuations is being stretched, new fluctuations are always being created which will then themselves be caught up in the expansion. By the end of inflation, there are small irregularities on a wide range of different length scales. Gravitational instability then acts on these small initial irregularities, and eventually, much much later, they can form galaxies and galaxy clusters.
This inflationary mechanism is currently the most popular model for the origin of structure, partly because it turns out to give mathematically simple predictions, but mainly because so far it offers excellent agreement with the real Universe, such as the microwave anisotropies just discussed. As I mentioned at the end of Section 13.5, there are presently quite a few different models for inflation, and typically their detailed predictions for the origin of structure are somewhat different. They therefore predict slightly different patterns of observed structures, hopefully different enough that one day we can use these structures to distinguish between inflation models observationally.

If the inflationary picture of the origin of structure is correct, a striking consequence is that all structures, including our own bodies, ultimately owe their existence to small quantum fluctuations occurring during the inflationary epoch. There can be no more dramatic example of the strong connection between microphysics and the large-scale Universe.