The Friedmann Equation

**Homogeneity** → constant density as a function of distance from center (varies with $t$)

Choose center (arbitrary, choose Milky Way) - galaxy at center is zero - cancels

The total force felt by a distant mass $m$ galaxy $r$ away from the center is from a mass $M$ at distance $r$:

\[ \frac{\dot{r}}{r^2} = -\frac{G M m}{r^3} \]

\[ \frac{1}{2} \dot{V} = -\int \frac{\dot{r}}{r} \, dr = -\frac{G M m}{r} \]

\[ U = T + V \]

\[ \dot{r}^2 = 2 \frac{m}{r} \frac{r^2}{2} - \frac{G M m}{r} \]

Isotropy → no preferred direction

$\dot{r}$ only depends on $\ddot{r}$, not $\dot{\theta}$ or $\dot{\phi}$

\[ \dot{V} = U = \frac{m}{2} \dot{r}^2 - \frac{G M m}{r} \]

Comoving Coordinates $\tilde{r} = a(t) \tilde{x}$

$\tilde{x}$ is a static grid (time independent!)

$\tilde{r}$ is an expanding grid

$a(t)$ depends only on time (not position)

\[ \ddot{x} = 0 \]

\[ \ddot{\tilde{r}} = a \ddot{x} + a^2 \dot{x}^2 = \ddot{x} \]
\[ U = \frac{1}{2} m a^2 x^2 - \frac{4\pi G}{3} \rho a^2 x^2 m \]

\[ = (\frac{1}{2} m a^2 - \frac{4\pi G}{3} \rho a^2 m)x^2 \] so \( U \propto x^2 \)

and

\[ (\frac{\dot{a}}{a})^2 = \frac{8\pi G}{3} \rho - \frac{kc^2}{a^2} \] where \( kc^2 = -\frac{2U}{m x^2} \) is a constant

The Friedmann Equation

Hubble Parameter \( H = \frac{\dot{a}}{a} \) varies with time

\[ 1 = \frac{8\pi G \rho}{3H^2} - \frac{kc^2}{a^2 H^2} = \Omega_0 + \Omega_k \]

Liddle will set \( c = 1 \) "natural units"

\[ k = c = k = 1 \]

\( k \) is the curvature constant of GR

\[ \Omega_k = -\frac{kc^2}{a^2 H^2} < 0 \] positive curvature \( (k > 0) \)

\[ \Omega_k = 0 \] flat

\[ \Omega_k > 0 \] negative

\( \rho \) is the density, which has 3 sources:

\[ \rho = \rho_m + \rho_r + \rho \]

\( \rho_m \) matter, \( \rho_r \) radiation, \( \rho \) dark energy

We will describe each density's evolution over time

by solving the Friedmann Equation, but first we need to explore the density of these "fluids"

Using \( \Omega_2 = \frac{8\pi G \rho}{3H^2} \) we will show

\[ \Omega_m = \frac{\Omega_m_0}{a^3} \quad \Omega_r = \frac{\Omega_r_0}{a^4} \quad \Omega_\Lambda = \Omega_\Lambda_0 \text{ const} \]