Newton's Law of Universal Gravitation
\[ \frac{\text{d} \vec{r}}{dt} = -\mathbf{G} \frac{\vec{m}_1 \vec{m}_2}{r^2} \]

"action at a distance"

\[ G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2} \]

\[ M_{\text{Sun}} = 1.99 \times 10^{30} \text{ kg} \]
\[ M_{\text{Earth}} = 5.97 \times 10^{24} \text{ kg} \]
\[ M_{\text{Moon}} = 7.35 \times 10^{22} \text{ kg} \]
\[ 2,360,000 \text{ s} \]
\[ T_{\text{Moon}} = 27.3 \text{ d} \text{ for one full circular orbit} \]
\[ T_{\text{Moon}} = 29.5 \text{ d} \text{ from full moon to full moon (lunar month)} \]

Newton could calibrate his theory (solve for GM)

Knowing \( g = \frac{4.8 \text{ m}}{s^2} \) on Earth's surface and Earth's radius from Eratosthenes \( R_E = 6371 \text{ Mm} \)

\[ g = \frac{GM}{r^2} \Rightarrow GM = gr^2 = 3.98 \times 10^{14} \text{ m}^3 \text{ s}^{-2} \]

\[ \vec{g} \] points towards away from Earth's center of Earth and the Moon's orbital radius from Hipparchus

\[ \sin \Theta = \frac{H}{r} \quad \text{for} \quad \Theta = (\frac{1}{10})^\circ \]

\[ \text{Src Alex} \Rightarrow \text{Syrene} \]

\[ \sin \Theta = \frac{\text{Src}}{R_E} \quad \sin \Theta = \frac{\pi}{50} = 7.2^\circ \]

\[ \Rightarrow \frac{R_E}{r} = \frac{\sin \Theta}{60} \Rightarrow r = 60R_E \]

Then the gravitational field at the Moon is

\[ \varphi_{\text{Moon}} = \frac{\varphi_{\text{Earth}}}{60^2} = 0.002762 \text{ m N kg}^{-1} \]

The centripetal acceleration is

\[ a = \frac{v^2}{r} = \left( \frac{2\pi}{T} \right)^2 r = 0.002709 \]