Paper 2: The Cosmological Constant Due 1:00PM April 12, 2013

While I may have consulted with other students in the class regarding this paper, the solutions presented here are my own work. I understand that to get full credit, I have to show all the steps necessary to arrive at the answer, and unless it is obvious, explain my reasoning using diagrams and/or complete sentences.

Name Signature:

In the second paper, you are to exhaustively work out and document the solutions to the Friedmann equation for a universe dominated by a cosmological constant. You should aim your paper at an undergraduate junior physics major (i.e., the writing level should be the same as that of Liddle).

- The paper should be 6-12 pages, double-spaced, in 11pt font, with 1 inch margins, including figures, emailed to me in PDF format no later than 1PM, Friday, April 12th. **No extensions will be given for this paper. Please manage your schedule accordingly.**

- Treat this paper like a very long problem set, solved and analyzed in the style of Liddle’s textbook. Use typeset equations and graphs to answer the various questions posed below. The quickest and most elegant way to do this would be via Mathematica.

- Try to treat all the problems below as a cohesive narrative, rather than answering them one-by-one in a disjointed fashion. Weave them together using your own language, try to make the presentation interesting and exciting. Above all, explain exactly what you are doing, much like a textbook would do.

- You do not have to answer all the problems in the order given. Choose any order that seems most natural to you.

- The paper should have an abstract and an introduction giving some background material relevant to the issue being discussed, before diving into answering the actual questions.

- Word-for-word copying from either the internet or your peers will result in a grade of 0. Make sure that the plots and equations are generated by you.

**Paper 2 Topic**

Imaging visiting a universe that contains only a cosmological constant, with vacuum density $\rho_\Lambda$ (which could be positive or negative). You enter the universe at time $t_0$, when the current Hubble constant is $H_0$. This universe contains no matter or radiation ($\rho_M = \rho_R = 0$). The current value of the vacuum density parameter is given by

$$\Omega_{\Lambda 0} = \Omega_\Lambda(t_0) = \frac{\rho_\Lambda}{\rho_c 0} = \frac{8\pi G \rho_\Lambda}{3 H_0^2}$$

To get full credit, you must address all of the following topics, in any order you wish:

- Write down the Friedman equation for this universe, eliminating $\rho_\Lambda$ and $k$ in favor of $\Omega_{\Lambda 0}$ assuming that $a(t_0) = 1$.

- Explain why even though $\rho_\Lambda$ is constant in time, $\Omega_\Lambda$ varies with time. There is one exception—which value of $\Omega_{\Lambda 0}$ gives $\Omega_\Lambda = \Omega_{\Lambda 0}$ for all time? Show mathematically why this is the case without explicitly solving for $a(t)$.

Please read both sides
• Depending on the value of $\Omega_\Lambda_0$, the universe will be open, closed, or flat. Write down which values give which type of universe.

• Still without solving the differential equation for $a(t)$, find the value of $a$ at which the universe switches direction (i.e., momentarily comes to a halt before going from expansion to collapse or vice versa). Show that there are three regimes—one where the universe does not switch direction, one where it goes from expansion to collapse, and one where it goes from collapse to expansion. Which values of $\Omega_\Lambda_0$ correspond to each of these choices?

• Solve the differential equation you wrote down above for $a(t)$, assuming that there is a big bang, i.e., your solution should give $a(0) = 0$. Show that this gives an ever-expanding universe for certain values of $\Omega_\Lambda_0$, and an oscillating, expanding-then-collapsing universe for certain other values. Show example plots of $a(t)$ for these two limiting values. Note that you have to be careful about the sign of $\dot{a}$ when considering the oscillating universe. Take care that $a$ is always positive.

• Solve the differential equation you wrote down above for $a(t)$, assuming that there is no big bang, i.e., your solution should use appropriate values of $\Omega_\Lambda_0$. Plot $a(t)$ for this case as well.

• Calculate the age of the universe, deriving a formula valid for all models which have a big bang. Plot the age versus $\Omega_\Lambda_0$.

• Comment on the meaning of “age” for the models without a big bang. Can you come up with a suitable substitute for “age”?

• Now use the solutions you work out above to plot the evolution of $\Omega_\Lambda$ as a function of time for different possible current values of $\Omega_\Lambda_0$. Is there a trend?