Weighing the Universe

Why we think Dark Matter isn’t made of baryons.
\[ H_0 = 100 \, h \, \text{km s}^{-1} \text{Mpc}^{-1}, \]  

and the final result from the Hubble Space Telescope Key Project gives

\[ h = 0.72 \pm 0.08, \]  

For a given value of \( H \), there is a special value of the density which would be required in order to make the geometry of the Universe flat, \( k = 0 \). This is known as the critical density \( \rho_c \), which we see is given by

\[ \rho_c(t) = \frac{3H^2}{8\pi G}. \]

Note that the critical density changes with time, since \( H \) does. Since we know the present value of the Hubble constant [at least in terms of \( h \) defined in equation (6.1)], we can compute the present critical density. Since \( G = 6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ sec}^{-2} \), and converting megaparsecs to metres using conversion factors quoted on page xiv, it is

\[ \rho_c(t_0) = 1.88 \, h^2 \times 10^{-26} \text{ kg m}^{-3}. \]
\[ H_0 = 100 \, h \, \text{km s}^{-1} \, \text{Mpc}^{-1}, \] (6.1)

and the final result from the Hubble Space Telescope Key Project gives

\[ h = 0.72 \pm 0.08, \] (6.2)

However, let us write that another way, since kilograms and metres are rather small and inconvenient units for dealing with something as big as the Universe. Let’s try measuring masses in solar masses and distances in megaparsecs. It becomes

\[ \rho_c(t_0) = 2.78 \, h^{-1} \times 10^{11} \, M_\odot/(h^{-1}\text{Mpc})^3. \] (6.6)

Suddenly this doesn’t look so small. In fact, \(10^{11}\) to \(10^{12}\) solar masses is about the mass of a typical galaxy, and a megaparsec more or less the typical galaxy separation, so the Universe cannot be far away (within an order of magnitude or so) from the critical density. Its density really must be around \(10^{-26}\) kg m\(^{-3}\).

So how close to critical density is real total density?

CMB anisotropies (from WMAP) suggest \(k = 0\) \(\Rightarrow\) \(\rho_0 \approx \rho_c\)

but densities of luminous and baryonic matter fall short...
The simplest thing we can do is look at all the stars within a suitably-large region. Stellar structure theory gives a good estimate of how massive a star is for a given temperature and luminosity. Provided we have looked in a large enough region, we get an estimate of the overall density of material in stars. This has been done by many researchers, and the answer obtained is that the density in stars is a small fraction of the critical density, around

\[
\Omega_{\text{stars}} \equiv \frac{\rho_{\text{stars}}}{\rho_c} \simeq 0.005 \rightarrow 0.01 . 
\]  

(9.2)

Not all of the material we are able to see is in the form of stars. For example, within clusters of galaxies there is a large amount of gas which is extremely hot and emits in the X-ray region of the spectrum, which I will discuss further below. Another possibility is that a lot of material resides in very low mass stars, which would be too faint to detect. Often discussed are brown dwarfs (sometimes called Jupiters), which are ‘stars’ with insufficient material to initiate nuclear burning. Objects with mass less than 0.08\(M_\odot\) are thought to be in this class. If for some reason there were a lot of objects of this kind then they could contribute substantially to the total density without being noticed, though this is not thought to be very likely on grounds of extrapolation from what we do know.

Nevertheless, there is a very strong reason to believe that conventional material cannot contribute an entire critical density. That evidence comes from the theory of nucleosynthesis — the formation of light elements — which will be discussed in Chapter 12. This theory can only match the observed element abundances if the amount of baryonic matter has a density

\[
0.016 \leq \Omega_B h^2 \leq 0.024. \quad h = 0.72 \pm 0.08 \quad \Omega_B \leq 0.059 
\]  

(9.3)

Recall from Section 2.5 that baryonic matter means protons and neutrons, and hence refers to the kinds of particle that we and our environment are made from.
In fact, there is considerable dynamical evidence that there is more than just the visible matter. The history of this subject is surprisingly old; in 1932 Oort found evidence for extra hidden matter in our galaxy, and one year later Zwicky inferred a large density of matter within clusters of galaxies, a result which has stood the test of time extremely well. The general argument is to look at motions of various kinds of astronomical object, and assess whether the visible material is sufficient to provide the inferred gravitational force. If it is not, the excess gravitational attraction must be due to extra, invisible, material.

One of the most impressive applications of this simple idea is to galaxy rotation curves. A galaxy rotation curve shows the velocity of matter rotating in a spiral disk, as a function of radius from the center. The individual stars are on orbits given by Kepler’s law; if a galaxy has mass \( M(R) \) within a radius \( R \), then the balance between the centrifugal acceleration and the gravitational pull demands that its velocity obeys

\[
\frac{v^2}{R} = \frac{GM(R)}{R^2}, \tag{9.4}
\]

which can be rewritten as

\[
v = \sqrt{\frac{GM(R)}{R}}. \tag{9.5}
\]

The mass outside the radius \( R \) contributes no gravitational pull, due to the same theorem of Newton’s we used to derive the Friedmann equation in Chapter 3.
\[ v = \sqrt{\frac{GM(R)}{R}}. \]

d\(M(R)/dR\) does not fall to zero where visible (luminous) matter suggests.
estimates suggest

\[ \Omega_{\text{halo}} \simeq 0.1. \]  \hspace{1cm} (9.6)

compared to

\[ \Omega_{\text{stars}} \equiv \frac{\rho_{\text{stars}}}{\rho_c} \simeq 0.005 \rightarrow 0.01. \]  \hspace{1cm} (9.2)
MAssive Compact Halo Objects (MACHOs)

MACHOs have been detected by gravitational lensing of stars in the Large Magellanic Cloud (LMC). The idea, illustrated in Figure 9.3, is to monitor LMC stars, which lie outside (or at least towards the edge of) the galactic halo. If there are invisible massive objects in the halo, and they happen to pass extremely close to our line of sight to the LMC star, then their gravitational field can bend and focus light from the star, temporarily brightening it. The only problem is that such events, called microlensing, are so rare that one has to monitor millions of stars in the LMC, every few days, for a period of years. Impressively, this became possible in the mid 1990s, and, to many people’s surprise, MACHOs were detected.

Figure 9.4 shows the brightness of a star in the LMC, monitored for around a year. Such a plot is known as a light curve, and we see a temporary brightening which lasted for around a month. The favoured explanation is gravitational lensing, rather than variability of the star, for several reasons. Firstly, the brightening only happened once, rather than periodically. Secondly, the brightening is the same in both red and blue light, whereas variable stars brighten differently at different wavelengths. And finally, the symmetric shape of the light curves matches that expected if an invisible gravitational lens were to pass in front of the star. The mass of these invisible objects is estimated as a little less than a solar mass. However, although present they appear to have insufficient density to completely explain the galactic halo. In fact, it may still be possible that both lens and star reside in the LMC itself.
MAssive Compact Halo Objects (MACHOs)

Figure 9.4 Light curves for a star in the LMC, obtained by the MACHO collaboration. The x-axis is in days with an arbitrary origin, while the y-axis shows the brightness of the star in red light and in blue light (the vertical units are a magnitude scale).
Gravitational Lensing
Gravitational Lensing
It turns out that we don't need to know how much an individual galaxy image has been lensed – we can instead work out the average lensing effect on a set of galaxies. To do so, cosmologists have to make a couple of assumptions: firstly, that all galaxies are roughly elliptical in overall shape, and secondly that they are orientated randomly on the sky, as shown in the left hand side of the figure below. In the presence of a lensing effect, we would expect that the galaxies in a patch of sky would appear to align themselves together slightly on the sky, as lensing stretches all their images in the same direction. In this way, any deviation from a random distribution of galaxy shape orientations is a direct measure of the lensing signal in that patch of sky. Weak lensing can thus be used to measure the gravitational lensing signal on any part of the sky.

http://www.cfhtlens.org/public/what-gravitational-lensing
estimates suggest

\[ \Omega_{\text{halo}} \approx 0.1. \] (9.6)

It is just about possible given present observations that this matter can be entirely baryonic, since this is marginally consistent with equation (9.3). However, many models based on low mass stars and/or brown dwarfs have been excluded and it is probably difficult to make up all of the halo with them. A popular alternative is to suggest that this density is in some new form of matter, which is non-baryonic and only interacts extremely weakly with conventional matter. This is reinforced by higher estimates for the matter density on larger scales discussed next. It is usually assumed that this dark matter lacks any dissipation mechanism able to concentrate it into a disk structure resembling that of the stars. If that is the case, then the dark matter should be in the form of a spherical halo, meaning a solid sphere with high density at the centre falling off to smaller values at large radii. The visible galactic disk and the globular clusters are embedded in this halo, as shown in Figure 9.2.

\[ 0.016 \leq \Omega_B h^2 \leq 0.024. \]
\[ h = 0.72 \pm 0.08 \]
\[ \Omega_B \leq 0.059 \] (9.3)
Galaxy clusters are the largest gravitationally-collapsed objects in the Universe, and as such are an ideal probe of the different kinds of matter. Because of their size, they should contain a fair sample of the material in the Universe, since there is no means of segregating different types of material as all is drawn in by gravity. The visible components of a galaxy cluster are in two main parts, seen in Figure 2.3 on page 6. There are the stars within the individual galaxies, and there is diffuse hot gas seen in X-rays which has been heated up through falling into the strong gravitational potential well of the cluster. It turns out that the baryon content of the latter is the greater, with about five to ten times more hot gas than stars. While only a small fraction of galaxies are in clusters, galaxy cluster formation is an ongoing process and those still to form ought to resemble those already present. The simplest assumption is that there is a considerable amount of cool gas between galaxies, which present technology does not allow us to detect, which will be heated if it becomes incorporated into a galaxy cluster. Note that the relative amounts of stars and hot gas in galaxy clusters are in excellent agreement with a comparison of the observed density of stars to the total baryon density inferred from nucleosynthesis.

\[
\Omega_{\text{stars}} \equiv \frac{\rho_{\text{stars}}}{\rho_c} \sim 0.005 \rightarrow 0.01. \quad (9.2)
\]

\[
0.016 \leq \Omega_B h^2 \leq 0.024, \quad h = 0.72 \pm 0.08 \quad \Omega_B \leq 0.059 \quad (9.3)
\]
Coma Cluster

http://www.chandra.si.edu/photo/2013/coma/
The hot gas can also be used to estimate the amount of dark matter present. Its high temperature gives it a substantial pressure, but it is confined to the galaxy cluster by gravitational attraction. However, the self-gravity of the gas alone does not provide enough attraction on its own, with the total mass of the cluster inferred to be around ten times larger than the gas mass. It is natural to assume that this extra attraction is given by dark matter, and if so the dark matter density must be around ten times larger than the baryon density given by nucleosynthesis. For example, data from the Chandra X-ray satellite have been used to give

$$\frac{\Omega_B}{\Omega_0} \approx 0.065 \ h^{-3/2},$$ \hspace{1cm} (9.7)

which using the nucleosynthesis constraint on $\Omega_B$ given above leads to

$$\Omega_0 \approx 0.3 \ h^{-1/2} \approx 0.35.$$ \hspace{1cm} (9.8)

This type of analysis indicates very directly that the matter density is dominated by dark matter, but that the dark matter density falls short of the critical density.
Further dynamical evidence for the existence of dark matter comes from the motions of galaxies relative to one another (i.e. the deviations from the cosmological principle). As with the rotation curves, the gravitational force exerted on galaxies by their neighbours depends on the total mass of the galaxies, whether it is visible or not. Galaxies possess relative motions, the peculiar velocities mentioned earlier, which allows one to estimate their mass under the assumption that their gravitational interaction is responsible for the motions, which are often termed bulk flows.

The analyses tend to be rather complicated, and indicate that the total density of matter in the Universe must obey

$$
\Omega_0 \geq 0.2 .
$$

As with the galaxy cluster determination of \( \Omega_0 \), this is well above the amount permitted by the nucleosynthesis constraint of equation (9.3). The conclusion therefore is not only that the Universe is largely composed of dark matter, but also that this dark matter must be non-baryonic rather than an invisible form of conventional material.

$$
0.016 \leq \Omega_B h^2 \leq 0.024 . \quad h = 0.72 \pm 0.08 \quad \Omega_B \leq 0.059
$$

So baryonic matter is about 10% of the total matter.
Bullet Cluster
Hot gas detected by Chandra in X-rays is seen as two pink clumps in the image and contains most of the "normal," or baryonic, matter in the two clusters. The bullet-shaped clump on the right is the hot gas from one cluster, which passed through the hot gas from the other larger cluster during the collision. An optical image from Magellan and the Hubble Space Telescope shows the galaxies in orange and white. The blue areas in this image show where astronomers find most of the mass in the clusters. The concentration of mass is determined using the effect of so-called gravitational lensing, where light from the distant objects is distorted by intervening matter. Most of the matter in the clusters (blue) is clearly separate from the normal matter (pink), giving direct evidence that nearly all of the matter in the clusters is dark.

The hot gas in each cluster was slowed by a drag force, similar to air resistance, during the collision. In contrast, the dark matter was not slowed by the impact because it does not interact directly with itself or the gas except through gravity. Therefore, during the collision the dark matter clumps from the two clusters moved ahead of the hot gas, producing the separation of the dark and normal matter seen in the image. If hot gas was the most massive component in the clusters, as proposed by alternative theories of gravity, such an effect would not be seen. Instead, this result shows that dark matter is required.

http://chandra.harvard.edu/photo/2006/1e0657/
One of the most remarkable observations in cosmology in recent years has been the first precision measurement of the geometry of the Universe using structures in the cosmic microwave background. I can only give a flavour of the result here, though more details will be given later in the book. As explained in Advanced Topic 5.4, structure formation scenarios predict a characteristic angular size, of around one degree, for features seen in the microwave background. The precise scale depends primarily on the geometry of the Universe, which tells us how the microwave photons travelled from their origin to our location.

The first precision measurement of the size of these features was announced by the Boomerang experiment in April 2000, followed swiftly by confirmation from the Maxima experiment. While the precise result does depend somewhat on assumptions, the simplest interpretation of those results is that the Universe is close to spatially-flat, with the total density (including any cosmological constant) lying within ten percent of the critical density. Further confirmation of this result has come from the WMAP satellite.
WMAP satellite’s map of CMB

http://wmap.gsfc.nasa.gov/media/121238/index.html
Figure A5.4 The radiation angular power spectrum as measured by the WMAP satellite, shown as the black dots. The solid line shows a theoretical prediction from their best-fit cosmological model, which fits the data extremely well. They define $C_\ell$ using the multipoles of $\Delta T$ itself rather than $\Delta T/T$, so their scale is $T_0^2 = (2.725 \text{ K})^2$ times that of Figure A5.3. I’ve drawn the figures so that their vertical axes match, but note that while Figure A5.3 uses a logarithmic $\ell$-axis, this plot uses a non-standard scaling along the $\ell$-axis chosen to display the observations evenly. [Figure courtesy NASA/WMAP Science Team.]
Type Ia supernovae data

Figure 4. The history of cosmic expansion, as measured by the high-redshift supernovae (the black data points), assuming flat cosmic geometry. The scale factor $R$ of the universe is taken to be 1 at present, so it equals $1/(1 + z)$. The curves in the blue shaded region represent cosmological models in which the accelerating effect of vacuum energy eventually overcomes the decelerating effect of the mass density. These curves assume vacuum energy densities ranging from 0.95 $\rho_c$ (top curve) down to 0.4 $\rho_c$. In the yellow shaded region, the curves represent models in which the cosmic expansion is always decelerating due to high mass density. They assume mass densities ranging (left to right) from 0.8 $\rho_c$ up to 1.4 $\rho_c$. In fact, for the last two curves, the expansion eventually halts and reverses into a cosmic collapse.
This microwave background data is especially powerful when combined with the data on supernova brightness described in Chapter 7. Both can be represented in the $\Omega_0-\Omega_\Lambda$ plane. A full discussion is made in the Advanced Topics, but I suggest you look now to Figure A2.4 on page 132. The two types of data favour long narrow regions crossing almost at right angles, and hence the region capable of fitting both data sets is extremely small. The favoured values are $\Omega_0 \simeq 0.3$ and $\Omega_\Lambda \simeq 0.7$, and the former is in excellent agreement with other measures of the matter density given earlier in this section. Note that the combination of direct measures of the dark matter density with the Boomerang results gives support to the cosmological constant independently of the supernova observations.

The **Wilkinson** Microwave Anisotropy Probe (WMAP) satellite’s data of the CMB suggest $k = 0 \Rightarrow \rho_0 = \rho_C \Rightarrow \Omega_0 = \rho_0 / \rho_C \simeq 1$

$$\Omega_0 = \Omega_{rel0} + \Omega_{m0} + \Omega_{\Lambda0}$$

Energy density of photons (CMB) and neutrinos
$$\Rightarrow \Omega_{rel0} = 4.15 \times 10^{-5} h^{-2} \leq 1.01 \times 10^{-4}$$

Type Ia supernova luminosity (distance) versus redshift (age) curves
$$d^2a / dt^2 > 0 \Rightarrow \Omega_{\Lambda0} > \Omega_{m0}$$

Dark Matter searches
$$\Rightarrow 0.2 \leq \Omega_{m0} \leq 0.35$$
$$\Omega_0 \approx 0.3 \text{ and } \Omega_\Lambda \approx 0.7,$$
- Luminous baryonic matter provides less than one percent of the total density.
- Dark baryonic material, probably mostly in the form of cool gas, is the dominant form of baryonic matter, overall making around four percent of the total density.
- There is around ten times as much non-baryonic dark matter as baryonic matter.
- The cosmological constant makes the largest contribution to the total density.
- All components added together give a density equal to, or at least close to, the critical density.

\[ \text{Nucleosynthesis:} \quad 0.025 \leq \Omega_B \leq 0.059 \]

\[ \text{Gravitational Forces:} \quad 0.2 \leq \Omega_{m0} \leq 0.35 \]

density of both dark and baryonic matter

\[ \text{Type Ia supernovae:} \quad \Omega_{\Lambda0} > \Omega_{m0} \]

\[ \text{COBE, Boomerang, MAXIMA, WMAP:} \quad \Omega_{k0} = 0 \]

\[ \text{Today}\]

- Dark Energy: 71.4%
- Dark Matter: 24%
- Atoms: 4.6%

http://map.gsfc.nasa.gov/universe/uni_matter.html
If Dark Matter isn’t baryonic, what is it?

Things we know exist: The particle which we know exists and yet whose properties are uncertain enough to allow it to be the dark matter is the neutrino. In the Standard Model of particle interactions the neutrino is a massless particle, and is present in the Universe in great abundance, being about as numerous as photons of light. If the Standard Model is extended to permit the neutrinos to have a small mass (a few tens of electron-volts), this would not affect their number density but they would have enough density to imply a closed Universe! The required density is comparable to, or perhaps slightly higher than, current experimental limits on the electron neutrino, but there are also the neutrinos associated with the muon and tau particles, and so they are more probable candidates.

Black Holes: A population of primordial black holes, meaning black holes formed early in the Universe’s history rather than at a star’s final death throes, would act like cold dark matter. However if they are made of baryons they must form before nucleosynthesis to avoid the nucleosynthesis bound of equation (9.3). Baryons already in black holes by the time of nucleosynthesis don’t count as baryons, as they are not available to participate in nuclei formation.

Things we believe might exist: Particle physicists regard supersymmetry as the most solidly-founded extension to standard particle theory, and it has the nice property of associating a new companion particle to each of the particles we already know about. In the simplest scenarios, the lightest supersymmetric particle (LSP) is stable and is an excellent cold dark matter candidate. Depending on the model the particle in question might be called the photino, or gravitino, or neutralino. They are also sometimes known as WIMPs — Weakly Interacting Massive Particles.
If Dark Matter isn’t baryonic, what is it?

Given the strength of the evidence that most of the matter in the Universe is dark matter, what can be done to discover it and study its properties? We’ve already discussed the detection of compact dark objects using microlensing, which can be used provided the masses are within a few orders of magnitude of a solar mass. But most of the favoured candidates for non-baryonic dark matter are elementary particles, whose masses are tiny fractions of a gram. Lensing certainly cannot be used to detect these.

The worst case scenario is if the dark matter particles interact with normal matter only through gravitational forces. If that is true then direct detection appears completely impossible: the gravitational force of an individual particle with say a proton mass is minuscule. The cumulative gravitational force of many such particles is measurable — that’s all the astrophysical dark matter evidence I’ve just discussed — but we want something more tangible.
If Dark Matter isn’t baryonic, what is it?

The best hope is if the dark matter particles interact not only gravitationally, but also through the weak nuclear force (hence the name Weakly Interacting Massive Particle or WIMP). Such interactions could, very reasonably, be feeble enough to have so far remained unobserved, but yet be within the realm of possible detectability. Supersymmetric particles in particular are thought to be potentially detectable if they indeed make up the dark matter.

Remember that the Universe is supposed to be full of these dark matter particles. Consequently, many of them will be streaming through your body at this very instant! Problem 11.2 will investigate this further for the case of neutrinos. You don’t notice them because their chance of interacting with you is so slight. But if you collect together enough material, and watch it for long enough, and the interaction rate is high enough, then every so often a dark matter particle will interact with a proton or neutron and give away the secret of its presence.